Quantum-to-classical transition in the presence of singularities

Aaron Goldberg[1,2], Duncan O'Dell[2]

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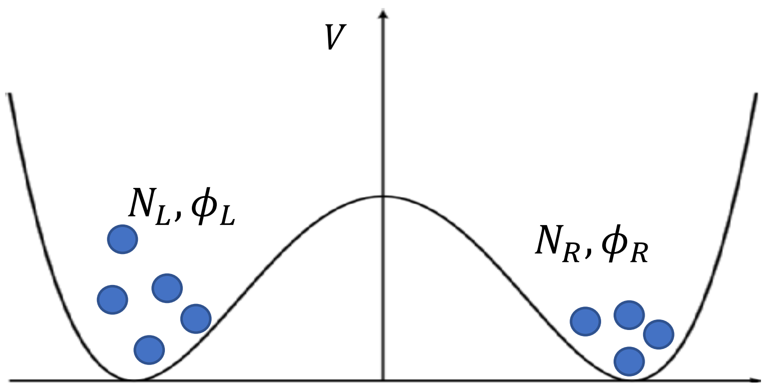
1. Background & motivation

Low temperatures → creation of Bose-Einstein condensates (BECs)

- Large numbers of particles occupy the same quantum state
- Macroscopic quantum effects
- Realized experimentally

2. Mean-field setup

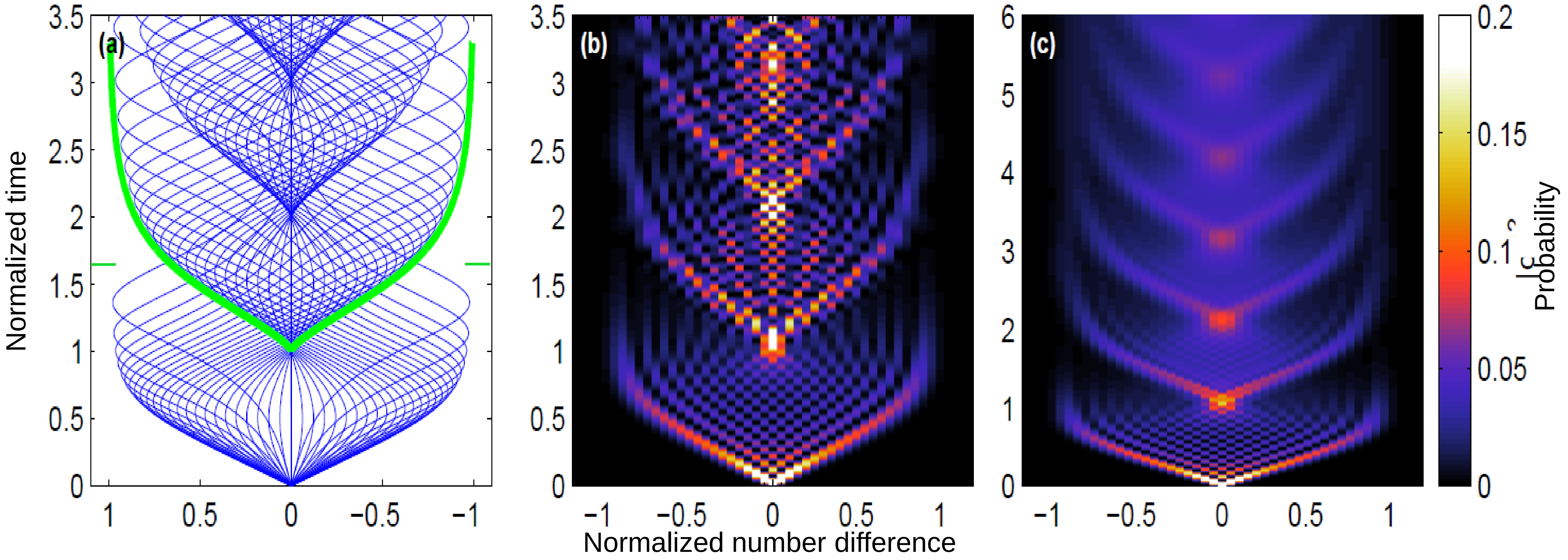


$$N = N_L + N_R$$
$$n = \frac{N_L - NR}{2}$$
$$\phi = \phi_L - \phi_R$$

$$H = \frac{E_C}{2} n^2 - E_J \sqrt{1 - \frac{4n^2}{N^2}} \cos\phi$$

$$\frac{\partial n}{\partial t} = -\frac{\partial H}{\partial \phi}, \frac{\partial \phi}{\partial t} = \frac{\partial H}{\partial n}$$

3. Dynamics for mean-field (a), quantum (b), and quantum with measurement (c)



4. Quantum treatment

$$\hat{H} = \frac{E_C}{2}\hat{n}^2 - \frac{E_J}{N}\left(\hat{a}_R^\dagger\ \hat{a}_L + \hat{a}_L^\dagger\ \hat{a}_R\right)$$

$$\hat{n} = \frac{\hat{a}_L^\dagger\ \hat{a}_L - \hat{a}_R^\dagger\ \hat{a}_R}{2}$$

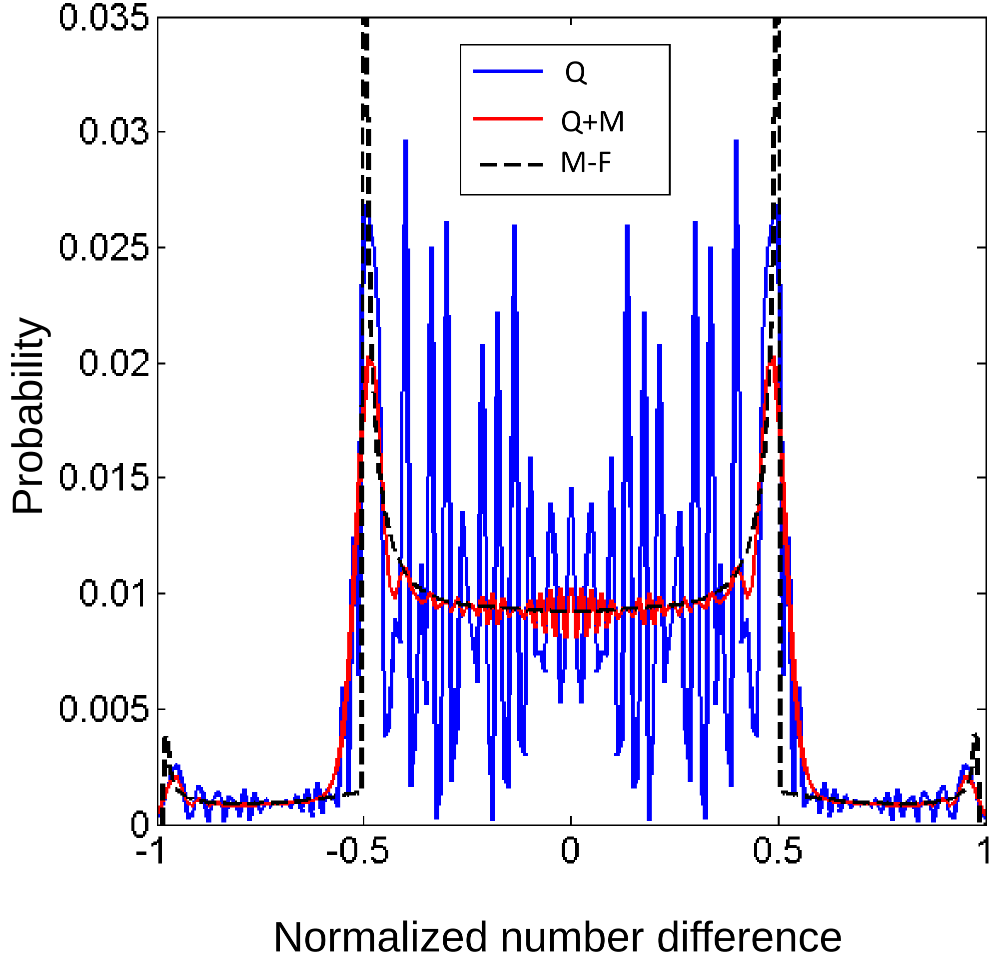
Schrödinger equation:

$$i\hbar\frac{d|\Psi >}{dt} = \hat{H}|\Psi >$$

5. Quantum + measurement

$$\dot{\rho} = -\frac{i}{\hbar}\left[\hat{H},\rho\right] - D\left[\hat{n},\left[\hat{n},\rho\right]\right]$$

6. Comparing singularities



7. Conclusions

- Quantized theory adds interference
- Continuous measurement damps quantum interference
- Mean-field behaviour recovered
- $N \to \infty$ limit reproduces diverging probabilities

4. Quantum treatments

$$\hat{H} = \frac{E_C}{2}\hat{n}^2 - \frac{E_J}{N}\left(\hat{a}_R^\dagger\ \hat{a}_L + \hat{a}_L^\dagger\ \hat{a}_R\right)$$

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1. Background & motivation

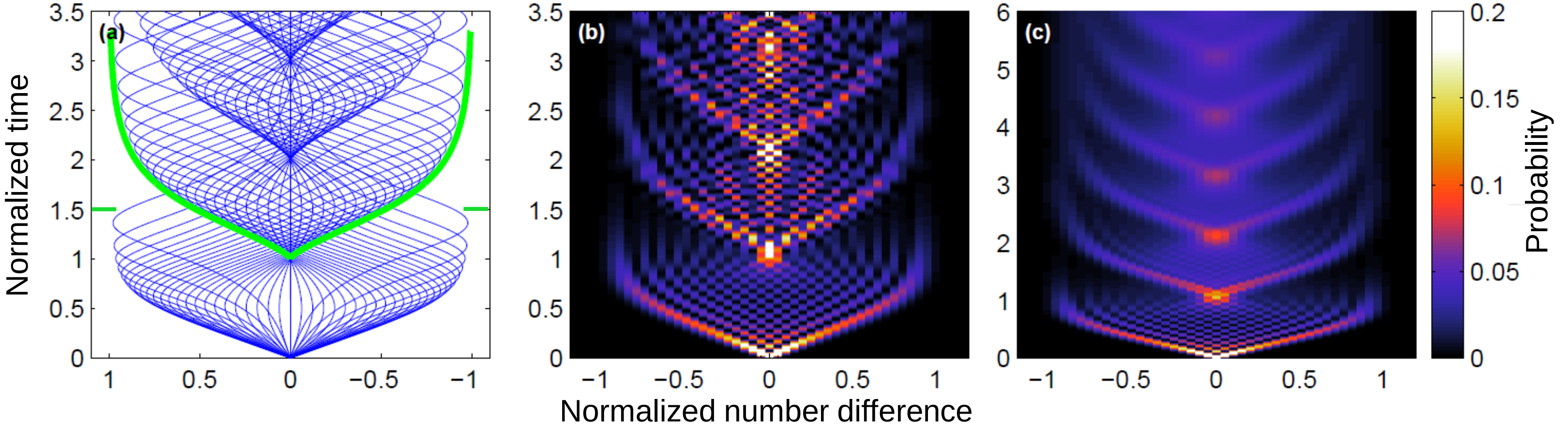
Low temperatures → creation of Bose-Einstein condensates (BECs)

- Large numbers of particles occupy the same quantum state
- Macroscopic quantum effects
- Realized experimentally [1]

Mean-field theory predicts diverging probabilities when two BECs are suddenly coupled together.

- Quantum theory regularizes singularities

3. Dynamics for mean-field (a), quantum (b), and quantum with measurement (c)



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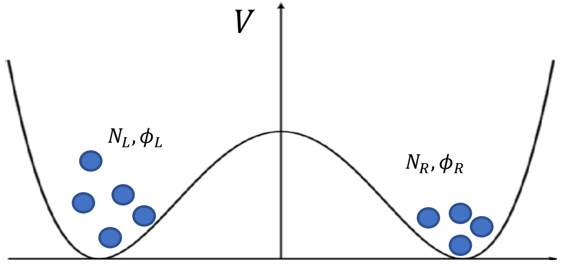

$$N = NL + NR$$

$$n = \frac{N_L - NR}{2}$$

$$\phi = \phi_L - \phi_R$$

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$$\hat{H} = \frac{E_C}{2}\hat{n}^2 - \frac{E_J}{N}\left(\hat{a}_R^\dagger\ \hat{a}_L + \hat{a}_L^\dagger\ \hat{a}_R\right)$$

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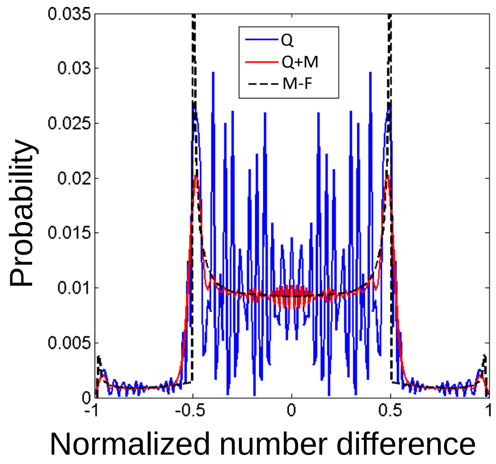
Schrödinger equation:

$$i\hbar\frac{d|\Psi>}{dt} = \hat{H}|\Psi>$$

5. Effect of measurement

$$\dot{\rho} = -\frac{i}{\hbar}\left[\hat{H}, \rho\right] - D\left[\hat{n}, \left[\hat{n}, \rho\right]\right]$$

6. Comparing singularities



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- Continuous measurement damps quantum interference
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- $N \to \infty$ limit reproduces diverging probabilities

[1] Davis, K.B., Mewes, M.-O., Andrews, M.R., van Druten, N.J., Durfee, D.S., Kurn, D.M. and Ketterle, W., 1995. Phys. Rev. Lett., **75**(3969).

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