Non-Gaussian effect in Ensemble Kalman filter

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Data Assimilation

Simulation

- $x_{t+1} = f(x_t)$
- We cannot get true initial condition
- Sensitivity to initial state

Observation

- $y_t = Hx_t + v$
- Observation of system is limited and noisy

Data Assimilation

Numerical Simulation $\oplus$ Bayesian estimation

- Dynamics of uncertainty $p(x_t)$
- Infinite dimensional dynamics

Kalman Filter: Parametric approach

Gaussian Assumption

$$
\begin{aligned}
p(x_t|y_{t-1},\cdots) &= \mathcal{N}({}^b\bar{x}_t, {}^bP_t), \\
p(x_t|y_t,\cdots) &= \mathcal{N}({}^a\bar{x}_t, {}^aP_t)
\end{aligned}
$$

Dynamics of Gaussian PDF

$$
\begin{aligned}
{}^b\bar{x}_{t+1} &= A_t {}^a\bar{x}_t \\
{}^a\bar{x}_t &= {}^a\bar{x}_t + K_t(y - H {}^b x_t) \\
{}^bP_{t+1} &= A_t {}^aP_t A_t^T \\
{}^aP_t &= (I - K_t H) {}^bP_t
\end{aligned}
$$

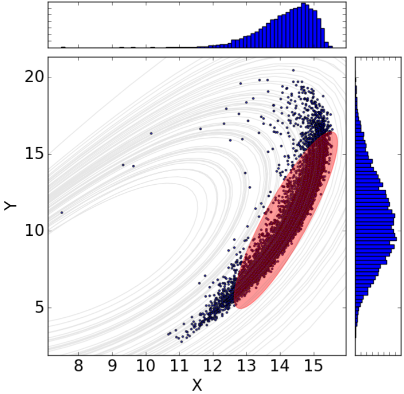
Advantage

- Finite dimensional
- Avoid curse of dimensionality

Disadvantage

- Really Gaussian assumption works?

Non-Gaussianity



quasi-Parametric Approach

- Ensemble Kalman filter
- Variational Bayes

Non-Parametric Approach

- Particle filter
- Markov-chain Monte-Carlo (MCMC)