Moment methods for radiation hydrodynamics

Oliver Völkel - Max Planck Institute for Astronomy, Heidelberg - Contact: voelkel@mpia.de

Rolf Kuiper - University of Tübingen

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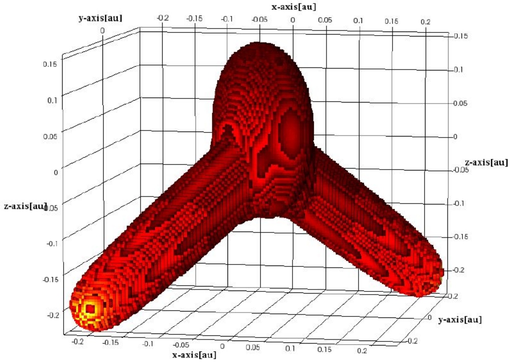
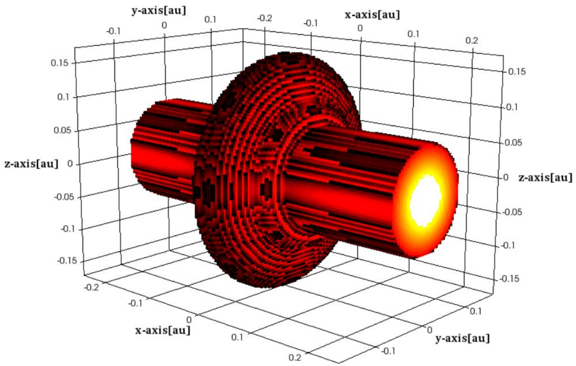
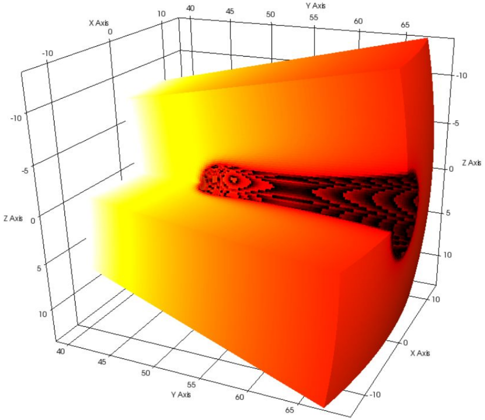






Visualization of radiation transfer using moment methods

Moment methods for radiation hydrodynamics

Outline and motivation

Coupling radiation with hydrodynamics is of crucial importance for numerous fields of research like astro-, plasma-, and laser-physics, but demands large computational resources due to its complex nature. Handling anisotropic radiation is a key necessity for fields like star formation or planet formation.

Numerically cheap models often fail to deliver the necessary physical complexity.
(No anisotropic radiation, see Fig.1)

Complex models like ray tracing or Monte carlo methods are often too expensive for dynamical (radiation-hydro) simulations.

The **M1 moment method** can handle anisotropic radiation at a competitive comutational cost.

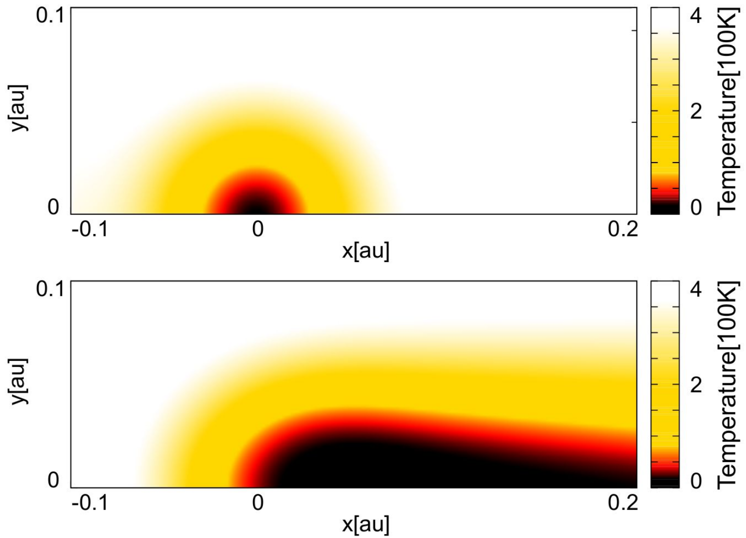


Fig.1
Irradiation coming from the left side across an opaque object. The numerically cheap module (flux limited diffusion-FLD) that cannot handle anisotropic radiation lets the light “flow” around the obstacle. The bottom module maintains a stable shadow behind the opaque object.

Moment methods for radiation hydrodynamics

The equations of radiation hydrodynamics

for moment methods

(in words)

$$\frac{\partial}{\partial t}\rho + \nabla\cdot(\rho\vec{v}) = 0$$

Conservation of mass

$$\frac{\partial}{\partial t}\rho\vec{v} + \nabla\cdot(\rho\vec{v}\otimes\vec{v}) + \nabla\cdot p = \rho a + \boxed{\frac{\kappa_R}{c}\rho F_r}$$

Conservation of momentum

$$\frac{\partial}{\partial t}E + \nabla\cdot(E+p)\vec{v} = \rho\vec{v}\cdot a - \boxed{\kappa_P\rho c(a_R T^4 - E_r)} + \boxed{\frac{\kappa_R}{c}\rho F_r\vec{v}}$$

Conservation of energy

$$\frac{\partial}{\partial t}E_r + \nabla\cdot F_r = \kappa_P\rho c(a_R T^4 - E_r)$$

Radiative energy density

$$\frac{\partial}{\partial t}F_r + c^2\nabla\cdot P_r = -\kappa_R\rho c F_r$$

Radiative flux

Adding radiative transfer has an effect on the evolution of the hydrodynamic medium.
Radiative pressure accelerates the gas and changes its momentum.
The energy equation changes by the kinetic energy of the accelerated gas and by radiative heating/cooling.

Hydrodynamics

Rad. transfer

Rad.-Hydro

Moment methods for radiation hydrodynamics

Solving radiation hydrodynamics (The reason for HPC)

The algebraic equation of the implicit M1 scheme consists of large numbers of self similar sparse tridiagonal matrices that are currently solved using the PETSc libraries and MPIs algebraic libraries.

The matrix A in the **one dimensional** system contains **4 tridiagonal submatrices**.

The **two dimensional** system contains **9 submatrices** which contain **multiple tridiagonal submatrices each** to account for the additional spatial dimension.

The system enlarges drastically for multiple dimensions, yet the multidimensional treatment of anisotropic radiation is the key feature of the M1 method.

$$\underbrace{\begin{pmatrix} E_rE_r & | & F_xE_r \\ --- & | & --- \\ E_rF_x & | & F_xF_x \end{pmatrix}}_{A} \underbrace{\begin{pmatrix} E_{ri}^{n+1} \\ --- \\ \tilde{F}_{ri}^{n+1} \end{pmatrix}}_{\alpha} = \underbrace{\begin{pmatrix} \Gamma_i^{E_{RHS}} \\ --- \\ \Gamma_i^{F_x,RHS} \end{pmatrix}}_{b}$$

$$E_rE_r = \begin{pmatrix}
\Gamma_{ic}^{E_rE_r} & \Gamma_{ir}^{E_rE_r} & 0 & \cdots & \cdots & \cdots & \cdots & 0 \\
\Gamma_{il}^{E_rE_r} & \Gamma_{ic}^{E_rE_r} & \Gamma_{ir}^{E_rE_r} & \ddots & & & & \vdots \\
0 & \Gamma_{il}^{E_rE_r} & \Gamma_{ic}^{E_rE_r} & \Gamma_{ir}^{E_rE_r} & \ddots & & & \vdots \\
\vdots & \ddots & \ddots & \ddots & \ddots & \ddots & & \vdots \\
\vdots & & \ddots & \ddots & \ddots & \ddots & \ddots & \vdots \\
\vdots & & & \ddots & \Gamma_{il}^{E_rE_r} & \Gamma_{ic}^{E_rE_r} & \Gamma_{ir}^{E_rE_r} & 0 \\
\vdots & & & & \ddots & \Gamma_{il}^{E_rE_r} & \Gamma_{ic}^{E_rE_r} & \Gamma_{ir}^{E_rE_r} \\
0 & \cdots & \cdots & \cdots & \cdots & 0 & \Gamma_{il}^{E_rE_r} & \Gamma_{ic}^{E_rE_r}
\end{pmatrix}, \quad \alpha = \begin{pmatrix} E_{r,0}^{n+1} \\ E_{r,1}^{n+1} \\ \vdots \\ E_{r,\mathrm{I_{max}+1}}^{n+1} \\ ---- \\ \tilde{F}_{x,0}^{n+1} \\ \tilde{F}_{x,1}^{n+1} \\ \vdots \\ \tilde{F}_{x,\mathrm{I_{max}+1}}^{n+1} \end{pmatrix}, \quad b = \begin{pmatrix} \Gamma_0^{E_{RHS}} \\ \Gamma_1^{E_{RHS}} \\ \vdots \\ \Gamma_{\mathrm{I_{max}+1}}^{E_{RHS}} \\ ---- \\ \Gamma_0^{F_x,RHS} \\ \Gamma_1^{F_x,RHS} \\ \vdots \\ \Gamma_{\mathrm{I_{max}+1}}^{F_x,RHS} \end{pmatrix}$$

Fig. 2
Matrix equation of the M1 scheme in one dimension.

HPC indispensable

Moment methods for radiation hydrodynamics

First results Voelkel et al. 2019 (subm.)

As seen in Fig.1(1 / 5) the M1 scheme can handle anisotropic radiation and therefore maintain stable shadows in radiation transfer simulations.

A planned and already tested application for the new module will be **planet disk interactions** during the formation of planets in opaque protostellar disks.

Simulations of these stages of planet formation using anisotropic radiation hydrodynamics can give important insights on the **formation and observability of forming planets around young stars**. A qualitative simulation of this setup can be seen in Fig. 3.

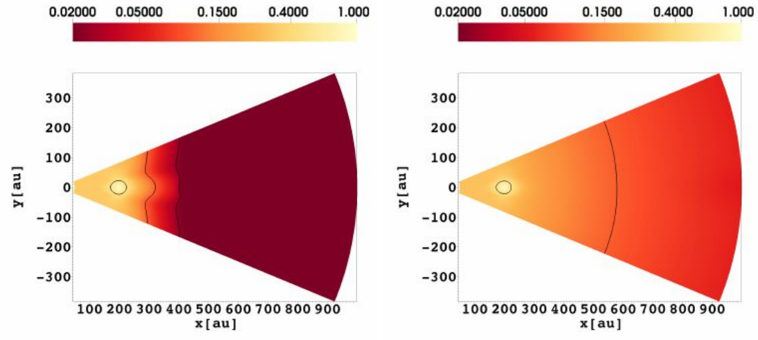


Fig. 3
Qualitative simulation of a radiating object in an opaque disk at equilibrium state. The left panel is computed using the M1 scheme and shows a stable shadow. The right panel was computed using the diffusive flux limited diffusion approximation and fails to maintain a shadow around the opaque disk.

Moment methods for radiation hydrodynamics

Summary and future challenges

We find that the M1 method delivers significant improvements for radiation hydrodynamical simulations by handling anisotropic radiation. This leads to stable shadows and a more realistic treatment of the radiative quantities in general.

These improvements are due to a more complex handling in the radiative transfer system than in the widely used flux limited diffusion approximation.

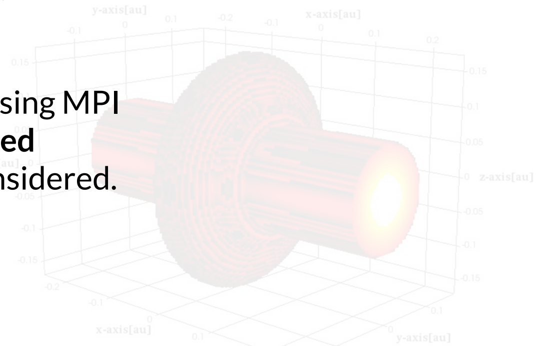
The computational cost of the M1 method exceeds that of the more simple FLD method (especially for multiple dimensions) but stays below that of more sophisticated methods like ray tracing or Monte Carlo methods.

Outlook and future work:

Future work lies in the **quantitative performance analysis and optimization** of the scheme in large scale computations to increase its usability for scientific studies.

A **direct cost comparison** between the M1 scheme and the FLD scheme.

The scheme is currently parallelized using MPI and PETSc, **but developing a GPU based version** of the code using CUDA is considered.



References: [1] D. Mihalas and B. Weibel-Mihalas. Foundations of Radiation Hydrodynamics. Dover Books on Physics. Dover Publications, 1984. ISBN 9780486409252. [2] M. González, E. Audit, and P. Huynh. HERACLES: a three-dimensional radiation hydrody- namics code. Astronomy and Astrophysics, 464:429–435, March 2007. [3] O.Voelkel. Moment methods for radiation hydrodynamics. Master Thesis 2017, University of Tübingen. [4] Voelkel, Kuiper, Gonzalez, in prep.