Operationally accessible entanglement of 1D spinless fermions

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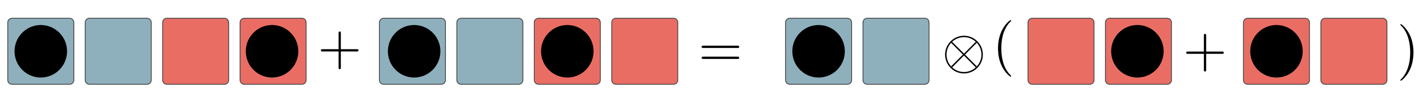
Abstract

The constituents of a quantum many-body system can be inextricably linked, a phenomenon known as quantum entanglement. Entanglement can be used as a resource for quantum computing and detecting phase transitions. It can be quantified via the von Neumann and Rényi entropies. Particle number superselection rules restrict the amount of entanglement that can be accessed as a resource. In this work, the accessible entanglement between spatial subregions of a 1D lattice of spinless fermions is quantified. Exact diagonalization results confirm the feasibility of the accessible entanglement entropy as a probe to detect quantum phase transitions.

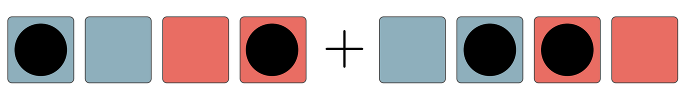
Quantum entanglement

A quantum many-body system can be split into smaller subsystems, or partitions. The system is entangled if the partitions are correlated non-classically. Entangled states cannot be factorized as a tensor product of the states of each partition:

Unentangled state: $|\Psi\rangle_{AB} = |\psi\rangle_A \otimes |\psi\rangle_B$



Entangled state: $|\Psi\rangle_{AB} \neq |\psi\rangle_A \otimes |\psi\rangle_B$



$\rightarrow$ Cannot be factored

Entanglement entropies

In an entangled system, performing a measurement on one of the partitions, will give information about the other. The more information is gained, the more entangled the system is. To quantify the information gain and, thus, the entanglement, the Rényi and von Neumann entanglement entropies are used:

$$S_\alpha(\rho_A) = \frac{1}{1-\alpha}\ln \mathrm{Tr}\rho_A^\alpha \quad \xrightarrow{\alpha \to 1} \quad S_1(\rho_A) = -\mathrm{Tr}\rho_A \ln \rho_A$$

Rényi — von Neumann

where ρ_A is the reduced density matrix of A, obtained by tracing out the B degrees of freedom of the full density matrix: $\rho = |\Psi\rangle\langle\Psi|$

Particle number superselection rules (SSR) restrict the amount of entanglement physically accessible as a resource. The accessible von Neumann entanglement is obtained by averaging the entanglement entropies over fixed local particle number sectors:

$$S_1^{\mathrm{acc}}(\rho_A) = \sum_n P_n S_1(\rho_{A_n}) \quad \text{where} \quad \rho_{A_n} = \frac{1}{P_n}\mathcal{P}_{A_n}\rho_A\mathcal{P}_{A_n}$$

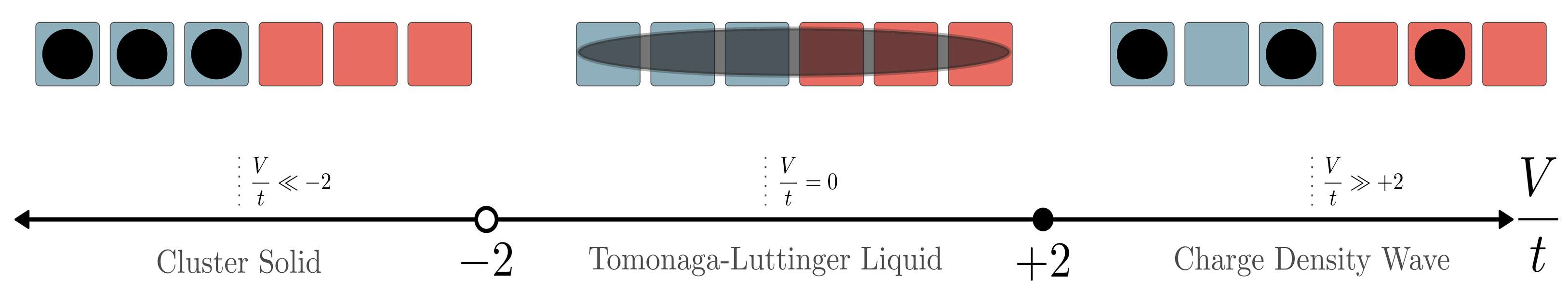
The t-V Model

This model describes the ground state of fermions hopping on a 1D lattice under periodic boundary conditions. The tunneling rate and interaction potential are tunable parameters, denoted by the letters t and V, respectively, hence the model's name. Its Hamiltonian is:

$$H = -t\sum_i (c_i c_{i+1}^\dagger + c_i^\dagger c_{i+1}) + V\sum_i n_i n_{i+1}$$

where $c_i^\dagger$ (c_i) creates (anihilates) a fermion on site *i* and n_i counts the number of fermions on *i*.

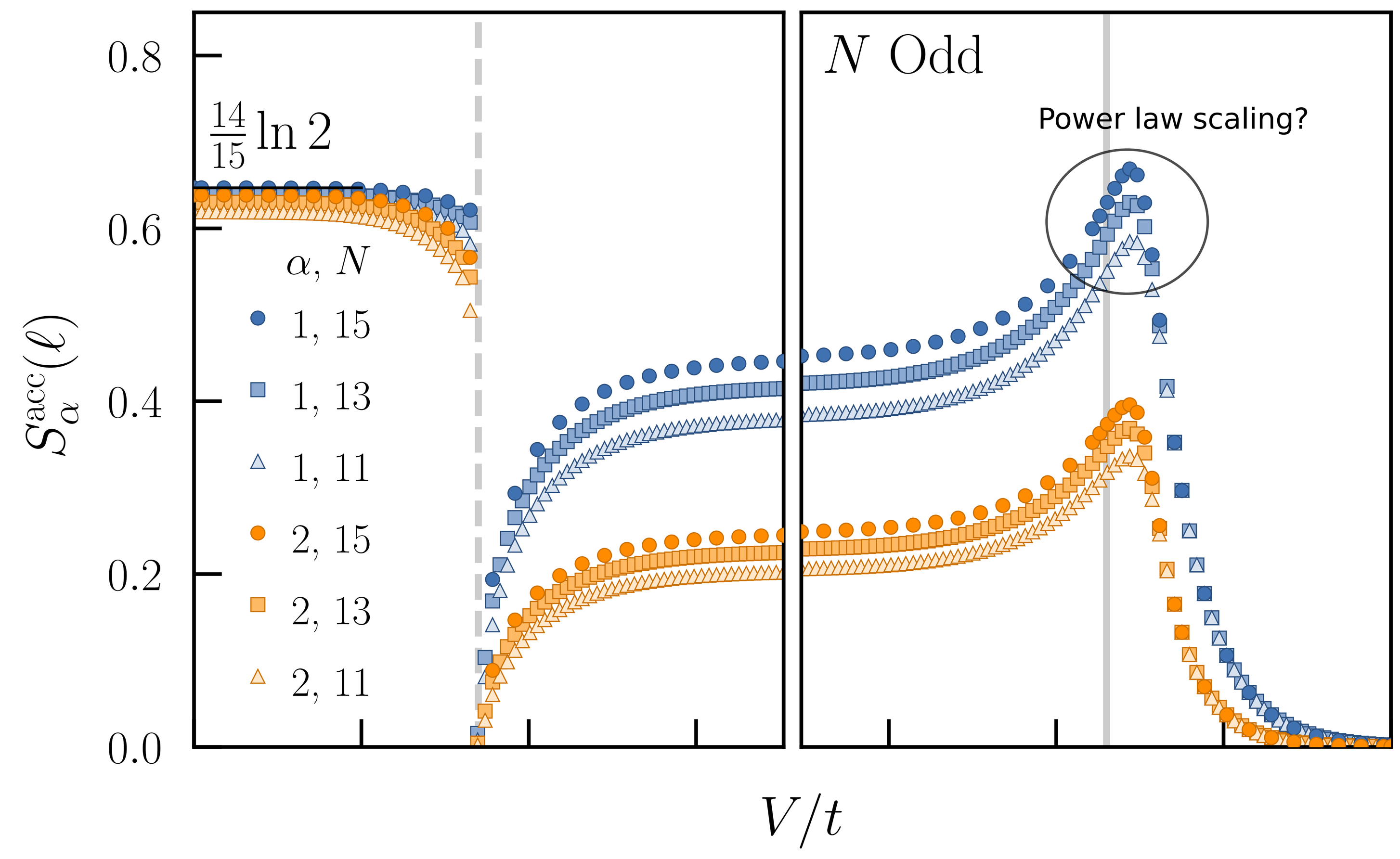
Phase diagram:



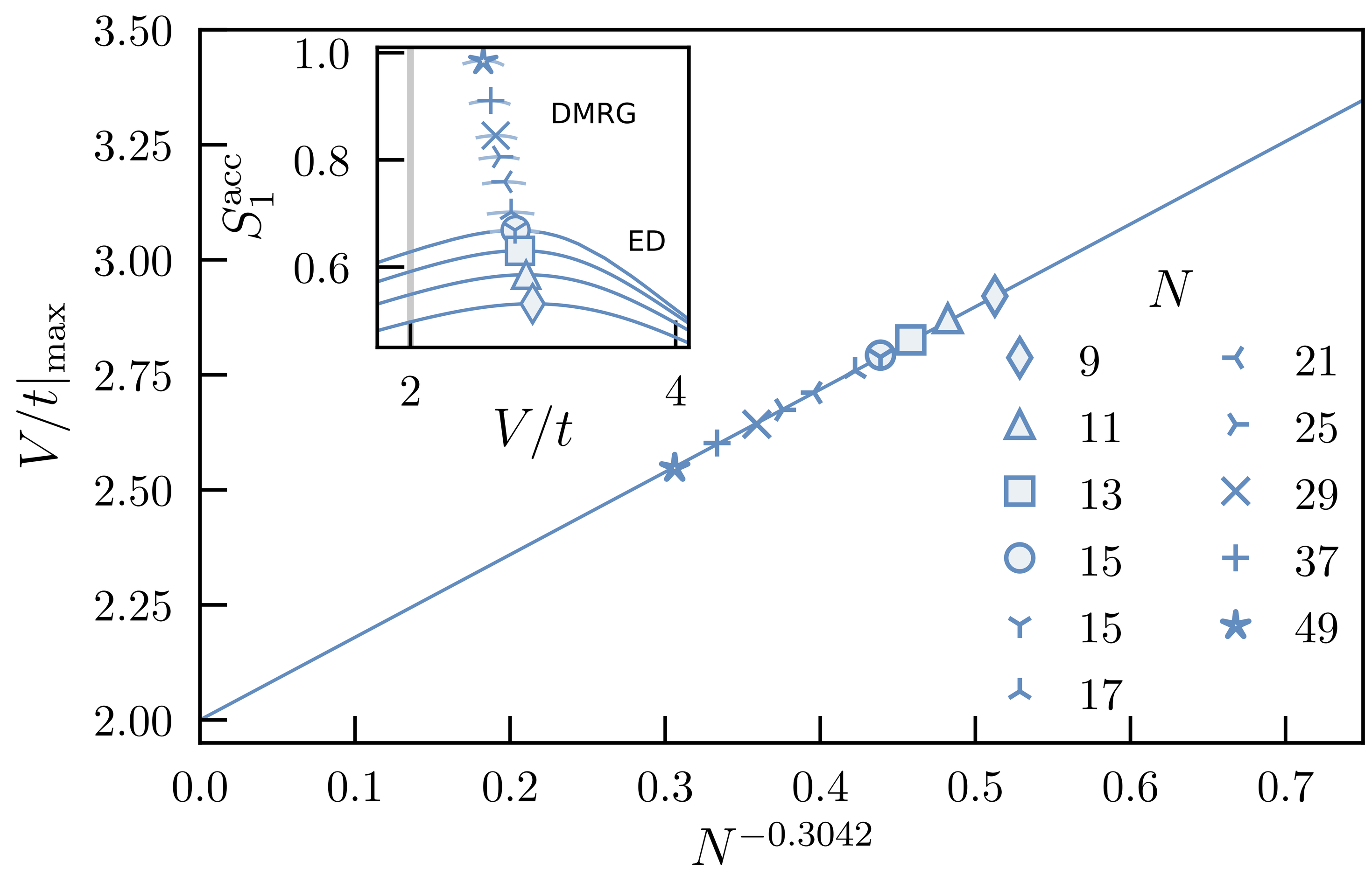
The above phase transitions can be detected by measuring the entanglement entropies as a function of interaction strength.

Exact diagonalization results

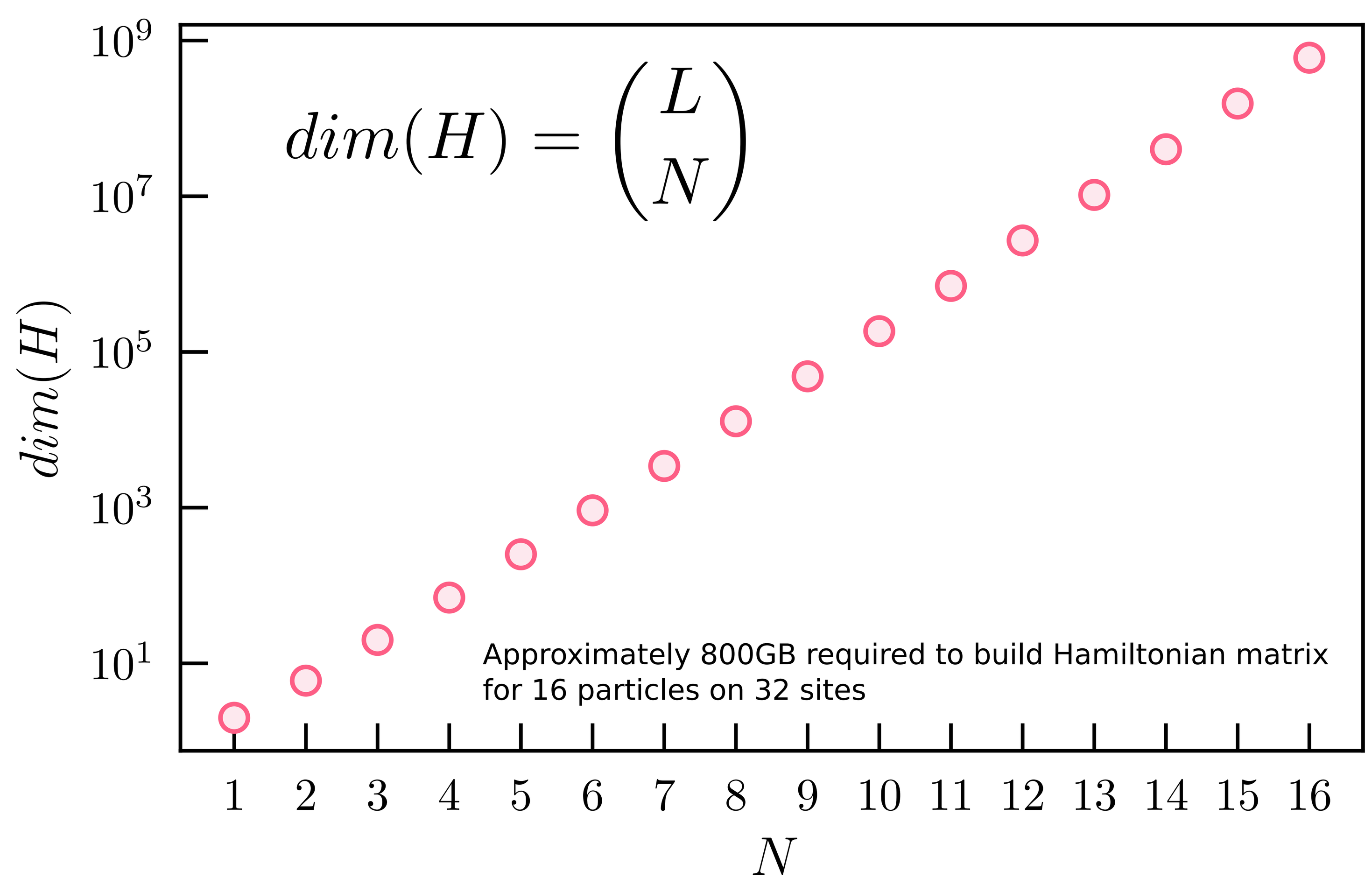
Accessible entanglement as a function of interaction strength:



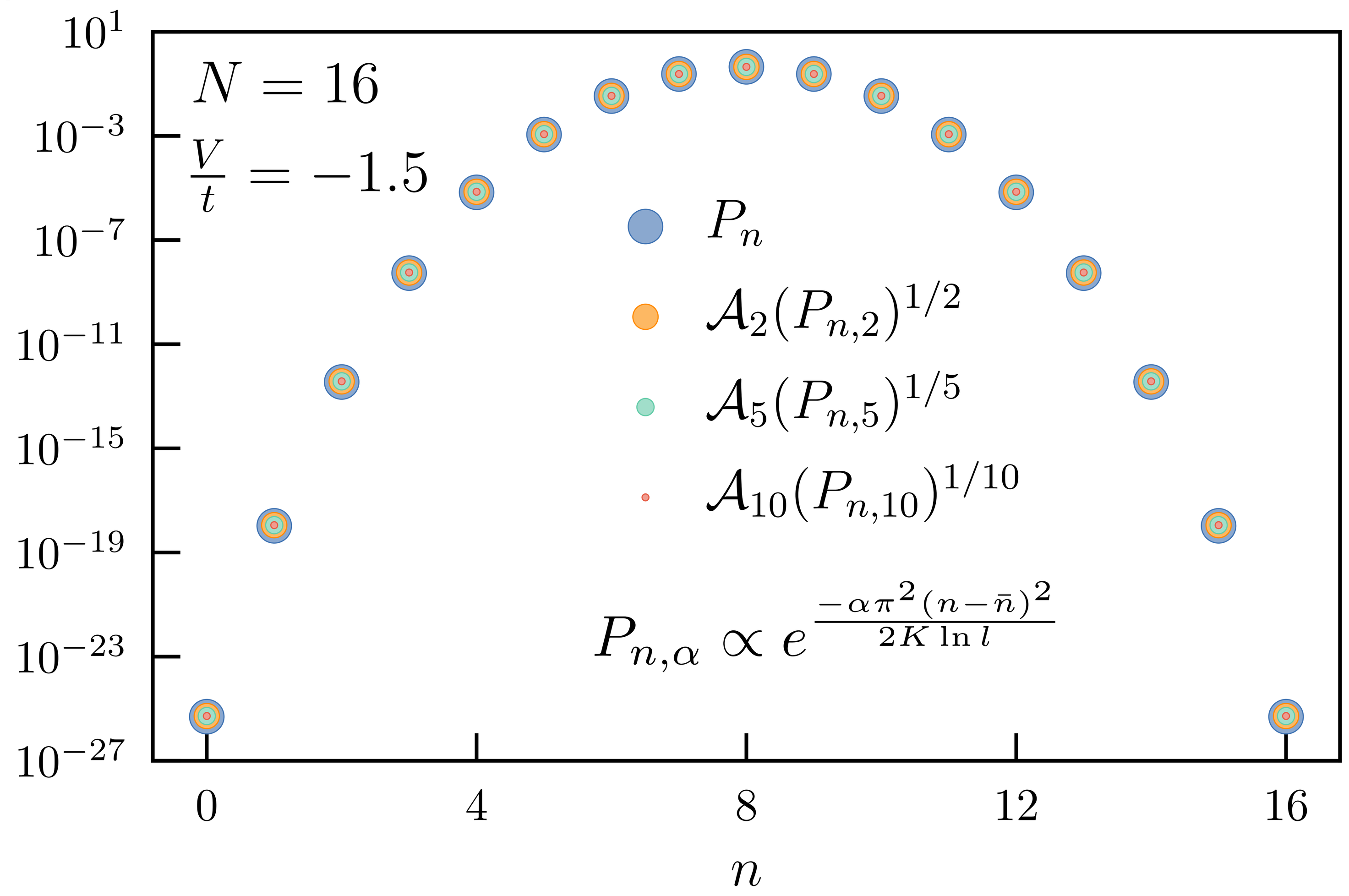
Scaling of accessible entanglement peak:



Hilbert space size dependence on system size:



Confirmation of Gaussian local particle number distribution in TLL phase:



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