A Divergence Free Augmented Immersed Boundary Method

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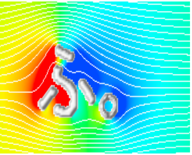

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Outline

Why Immersed Boundary

- Moving boundary problems and heat transfer is of great relevance to many engineering applications.
- The conventional methods (ALE, Moving Mesh, etc.) increase the computational cost of per-grid-point operation.
- Immersed boundary (IB) methods provide an alternative approach for simulating FSI and Heat Transfer problems involving complex geometries and arbitrarily large deformations
- Fast grid generation without the need to conform the grid to the fluid–structure interface, which may be undergoing arbitrarily large deformation and/or possess a complex shape.
- The use of efficient flow solvers for solving the governing equations on the stationary grid

The Mathematical Formulation of the New Immersed Boundary Method

- With the current formulation both velocity-vorticity and pressure-velocity forms of the incompressible *Navier-Stokes* equations may be employed.
- The domain of the computation Ω is a simple rectangular domain in which the obstacle is represented by the force term $\boldsymbol{f}$ imposing on the subdomain Ω_s.
- A fractional step method is employed for time integration. Space discretization is implemented by either pseudo-spectral or 2^{nd} order central finite difference.

$$\begin{cases} \frac{\partial \boldsymbol{\omega}}{\partial t} = (\boldsymbol{v} \cdot \nabla)\boldsymbol{\omega} + \nu \nabla^2 \boldsymbol{\omega} - \boldsymbol{g}\beta \nabla T + \boldsymbol{f} \quad \text{for} \quad \boldsymbol{\omega} \in \Omega \\ \nabla^2 \boldsymbol{u} = \boldsymbol{\omega} \quad \text{for} \quad \boldsymbol{u} \in \Omega \end{cases}$$

$$\begin{cases} \frac{\partial \boldsymbol{u}}{\partial t} = \boldsymbol{u} \cdot \nabla \boldsymbol{u} - \nabla p + \nabla^2 \boldsymbol{u} + \boldsymbol{f} \quad \text{for} \quad \boldsymbol{u} \in \Omega \\ \nabla \cdot \boldsymbol{u} = 0 \end{cases}$$

- By the basis $\mathbf{P} = \{\phi_1, \ldots, \phi_n\}$ such that $span\{\phi_1, \ldots, \phi_n\} = \pi_m(\mathrm{R}^d)$ the optimal coefficient vector solution, $\mathbf{c}$, for the least square problem (our approximation for the field variable)

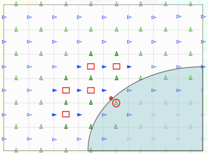
$$\min_{p \in \pi_m(\mathrm{R}^d)} \left\{ \sum_{j=1}^{N} \left[(\mathbf{u}(x_j) - \mathbf{p}(x_j))^2 + \epsilon_1 (\mathbf{f}_j - \mathcal{L}\mathbf{p}_j)^2 \right] W(\|x - x_j\|) \right\},$$

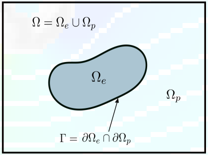
is obtainable as

$$\mathbf{c}^{vec} = \mathbf{M}^{vec}(x)^{-1} \sum_j \mathbf{P}_j^T W^{vec}(\|x - x_k\|)\, \mathbf{u}^{vec},$$

where,

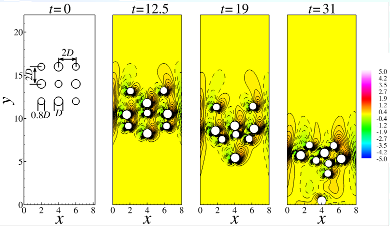
$$\mathbf{M}^{vec}(x) = \sum_k \mathbf{P}_k^T W^{vec}(\|x - x_k\|)\, \mathbf{P}_k.$$





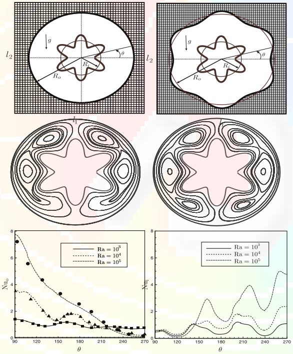
Fluid–Rigid Structure Interactions

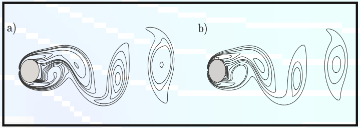
- Fluid interacting with a flat plate with zero thickness has some challenging features that now can be tackled.
- Falling colliding cylinders interacting with fluid and possesses different physical phenomena.

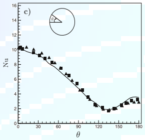


Natural and Forced Heat Convection

- The accuracy of the heat convection simulation particularly the natural heat convection are highly sensitive to the mass conservation and time integration accuracy.







Future Plans & References

★ **Plans:**

- ✓ Adapting the higher order discretizasion and interpolation methods
- ☐ Using turbulence model to tackle higher Reynolds problems
- ☐ To equip the solver with adaptive mesh
- ☐ Adding some structural models to facilitate the elastic structure modeling

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