Bifidelity Data-assisted Neural Networks In Nonintrusive Reduced-order Modeling

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Objective

When there's a high-fidelity model and a cheap low-fidelity model available, we aim to develop a non-intrusive method to produce an accurate approximation of the high-fidelity solution with affordable online cost, particularly for nonlinear problems.

POD

Let $S_h = [u_h(z_1), u_h(z_2), \ldots, u_h(z_P)]$ be a snapshot matrix for the high-fidelity model. Proper Orthogonal Decomposition (POD) is given by the singular value decomposition

$$S_h = U_h \Sigma_h W_h^\top. \tag{1}$$

The first r columns of U_h are chosen to form the POD basis V_h of the reduced space $\mathbb{V}_h$ and the POD coefficients are computed by a projection onto $\mathbb{V}_h$:

$$c_h(z) = V_h^\top u_h(z). \tag{2}$$

The reduced solution is represented by

$$u_r(z) = V_h c_h(z). \tag{3}$$

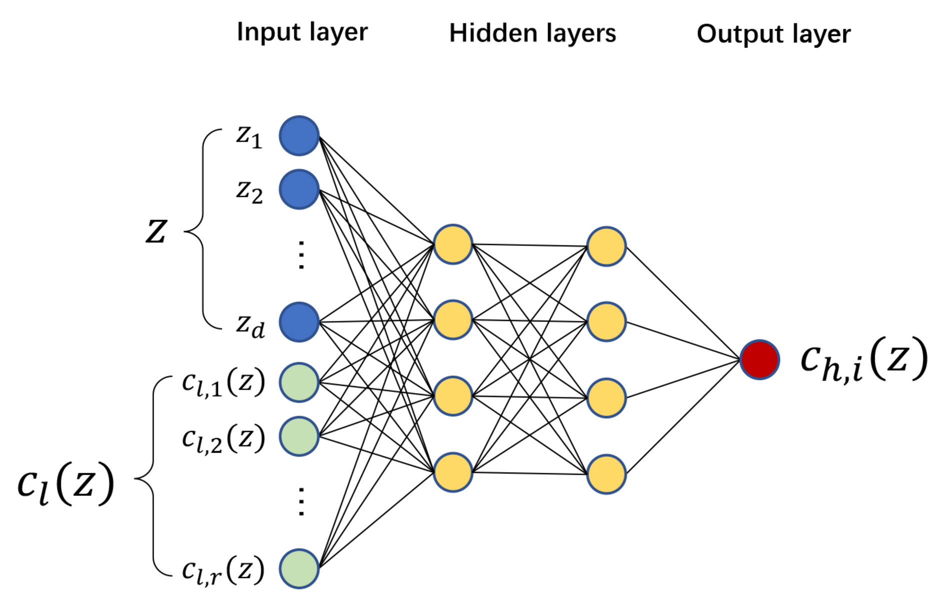
For traditional nonintrusive RB methods, the projection coefficients are obtained via interpolation over the parameter domain. However, this approach is not robust due to the strong nonlinearity [1].

BiFi-NN

To improve the approximation power, we propose to include features extracted from the low-fidelity model, besides the original parameter z [3].

Offline:

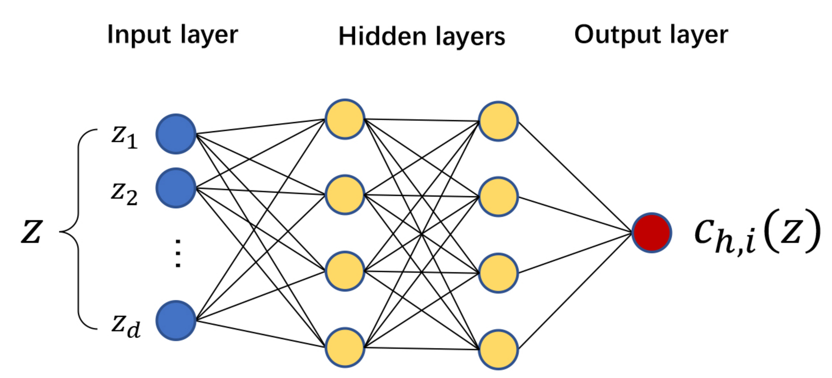
1. Sample a collection of parameters $\Gamma = \{z_1, z_2, \ldots, z_M\} \subset I_z$. For each $z_j \in \Gamma$, run the high-fidelity model $u_h(z_j)$ and the low-fidelity model $u_l(z_j)$.
2. Compute the POD coefficients $c_l(z)$ and $c_h(z)$ for both fidelities by projection.
3. For $i = 1, \ldots, r$, train a network $\Phi_i(x;\theta)$ where the input is the combined feature $x = (c_l(z), z)$ and the output is the i^{th} component of $c_h(z)$.



Online:

1. Run the low-fidelity model $u_l(z^*)$ for the given z^*.
2. Compute the low-fidelity POD coefficients $c_l(z^*)$ by projection.
3. For $i = 1, \ldots, r$, evaluate the pre-trained network Φ_i at the combined feature $x^* = (c_l(z^*), z^*)$ to the approximation of i^{th} component of $c_h(z^*)$.
4. Construct the approximated high-fidelity solution.

Remark: We also provides a modified POD-NN structure as a baseline method [2].

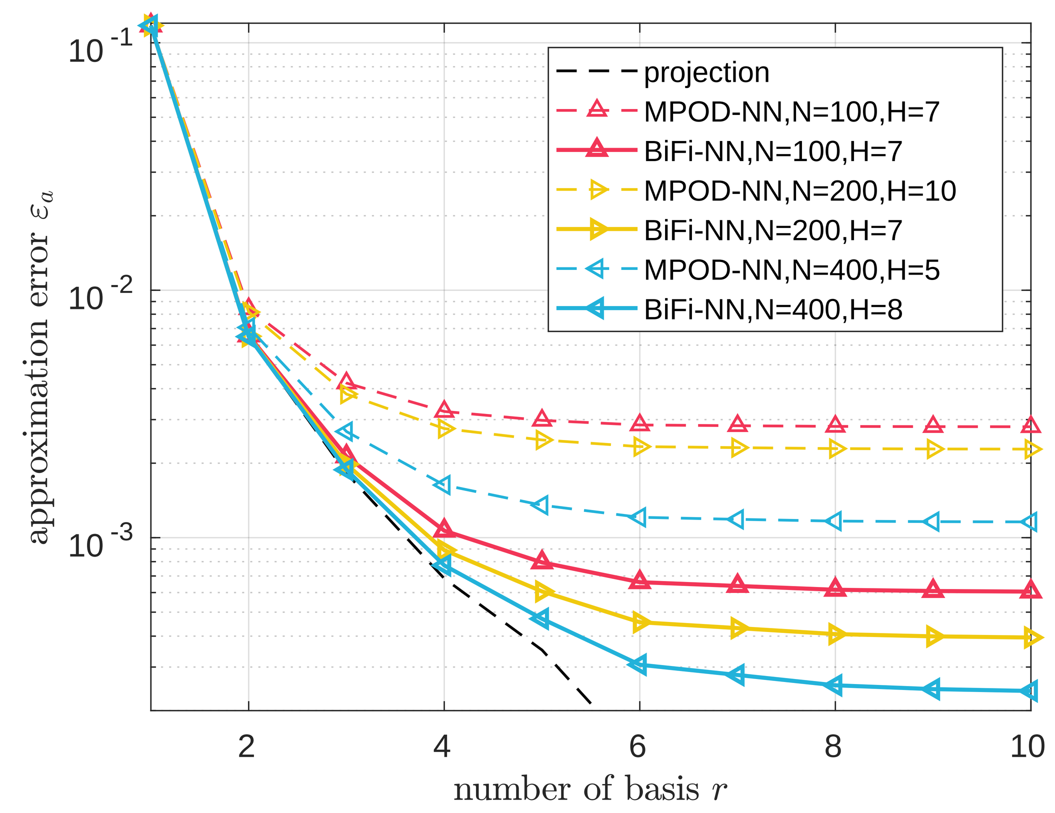


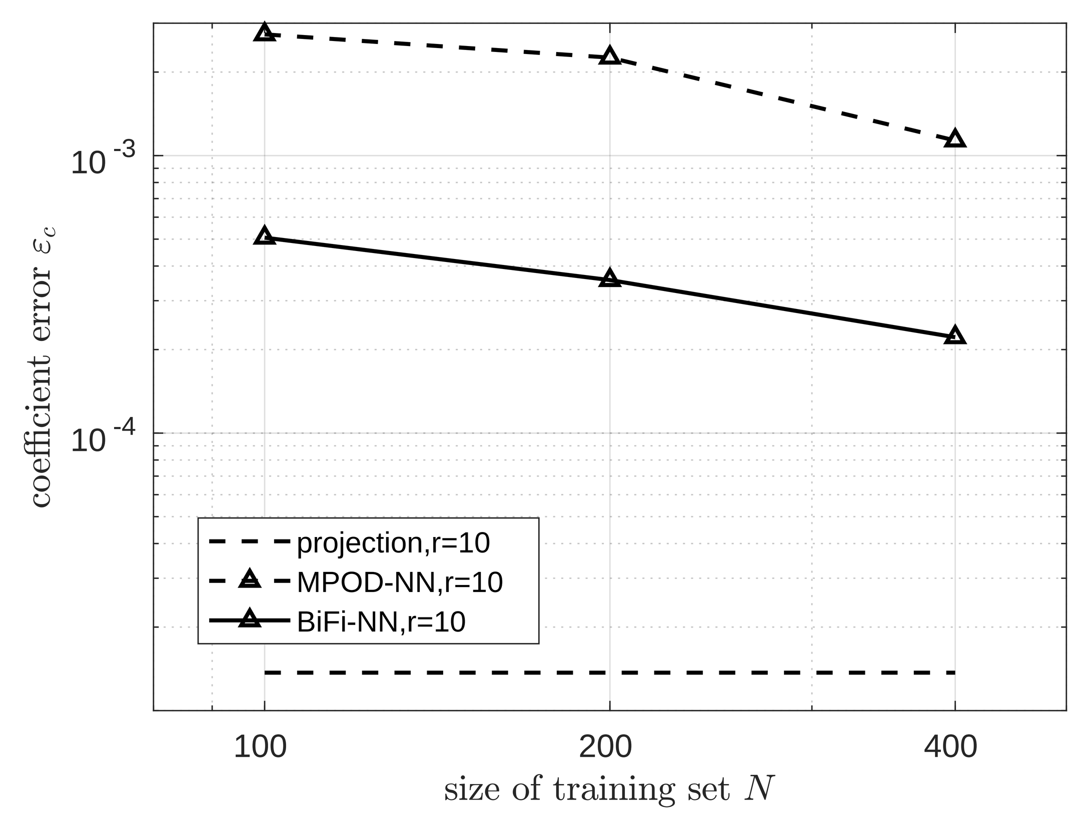
A 2D nonlinear elliptic equation

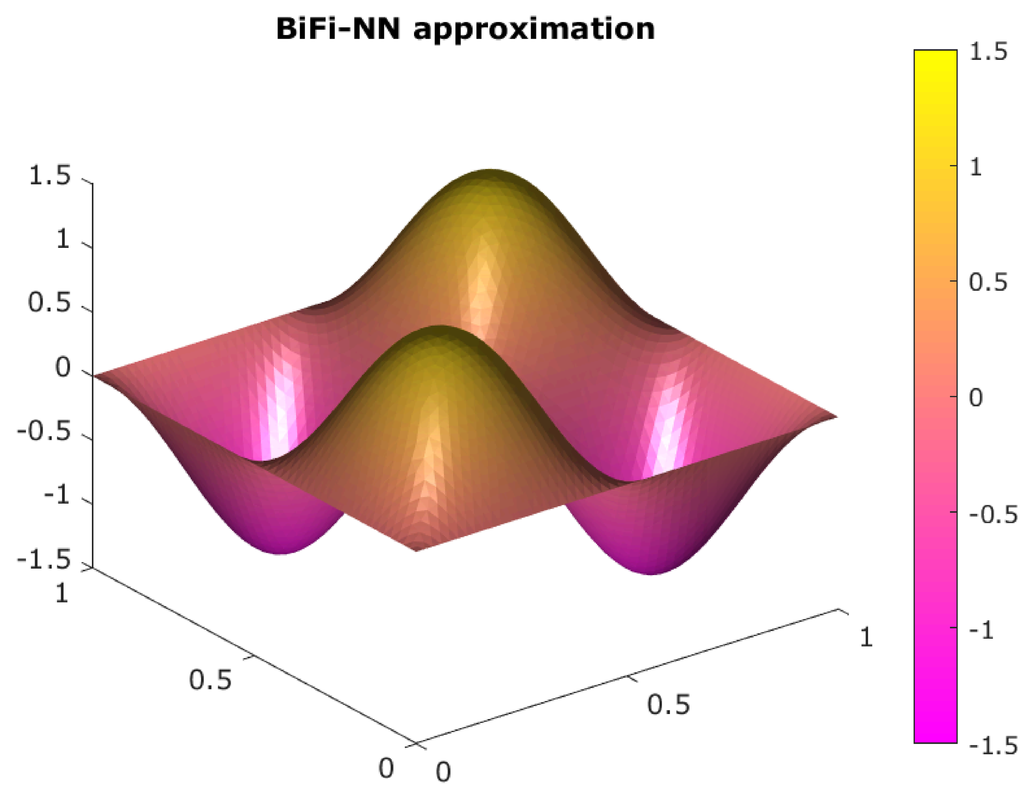
Consider a parameterized 2D nonlinear elliptic equation with parameter $\mu = (\mu_1, \mu_2) \in [0.01, 10]^2$:

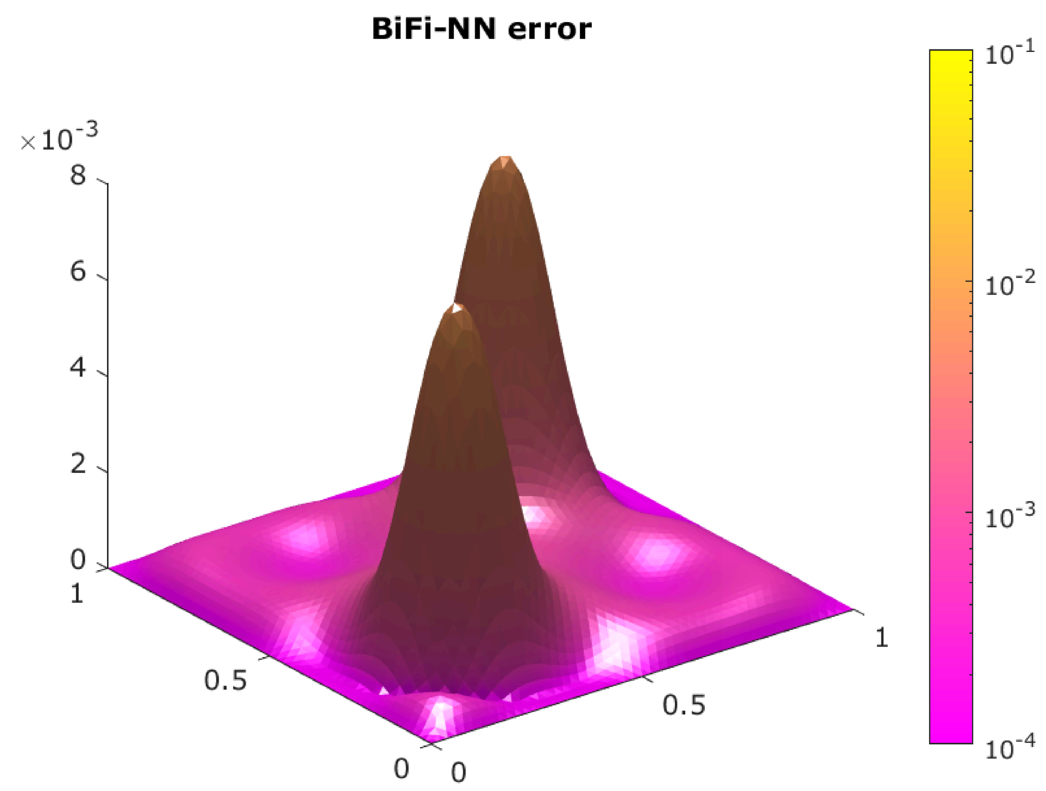
$$-\Delta u(x,y) + s(u(x,y);\mu) = 100 \sin(2\pi x)\sin(2\pi y), \quad s(u;\mu) = \frac{\mu_1}{\mu_2}(e^{\mu_2 u} - 1), \tag{4}$$

with a homogeneous Dirichlet boundary condition. The spatial domain is $(x,y) \in \Omega = (0,1)^2$. We use 135 P_1 elements to solve the low-fidelity model, and 2960 elements for the high-fidelity model.









- The proposed BiFi-NN method is a nonintrusive method.
- It is able to produce a reasonable good approximation of the high-fidelity solution.
- The online cost of BiFi-NN mainly depends on the low-fidelity model.

Reference

[1] Volker Barthelmann, Erich Novak, and Klaus Ritter. High dimensional polynomial interpolation on sparse grids. *Advances in Computational Mathematics*, 12(4):273–288, 2000.

[2] Jan S Hesthaven and Stefano Ubbiali. Non-intrusive reduced order modeling of nonlinear problems using neural networks. *Journal of Computational Physics*, 363:55–78, 2018.

[3] Chuan Lu and Xueyu Zhu. Bifidelity data-assisted neural networks in nonintrusive reduced-order modeling. *submitted*, 2019.