

Black holes
in computers

Jens F.
Mahlmann

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References

Black holes in computers

Relativistic jets in progenitors of gamma-ray-bursts



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Force-free magnetospheres in computers

Focusing on electrodynamics around compact objects

Goal: Scalable code for the simulation of *force-free* magnetospheres on the dynamical metric of *compact objects* (e.g., neutron stars or black holes).

Physics: In the limit of high field energy (low particle inertia), General Relativistic magnetohydrodynamics (GRMHD) reduces to *General Relativistic force-free electrodynamics (GRFFE)*.

[1] Full Maxwell's equations evolution

(Komissarov, 2002, 2004, 2007; Paschalidis and Shapiro, 2013)

$$\nabla_\nu F^{\mu\nu} = \textcolor{teal}{J}^\mu \quad \nabla_\nu {}^*F^{\mu\nu} = 0$$

[2] Energy flow evolution

(McKinney, 2006; Paschalidis and Shapiro, 2013;
Etienne et al., 2017)

$$\nabla_\mu T^\mu_\nu = 0 \quad \nabla_\nu {}^*F^{\mu\nu} = 0$$

- *Explicit* methods on *structured* grids (well suited for hyperbolic, non-stiff equations)
- *Conservative* schemes require computing a unique *flux* per numerical zone (exact and HLL *Riemann solvers*)
- Cover a variety of time and length scales (e.g., at *current sheets*). This requires adaptivity of the *mesh*, highly accurate regions, and parallel scalability.

Simulations of Blandford/Znajek process

Setting up a generic GRFFE problem

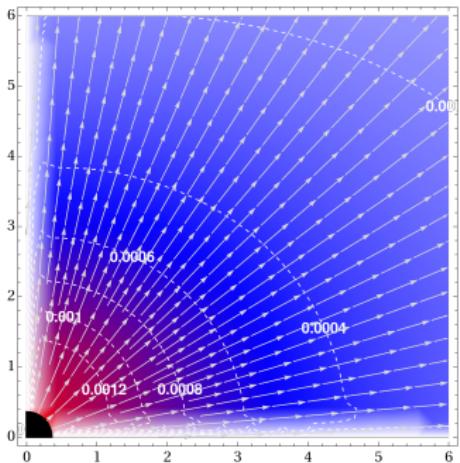


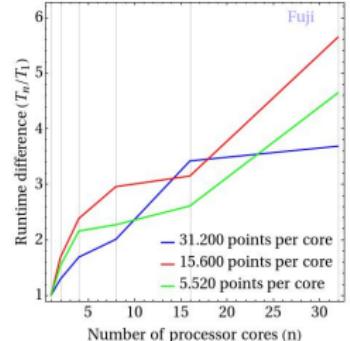
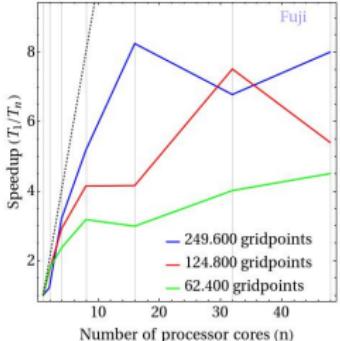
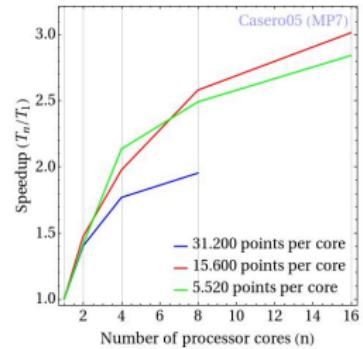
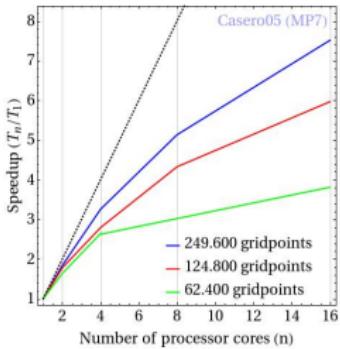
Figure: Visualization of the mag. field (\mathbf{B}) initial data around the BH (mass $m = 1$, spin $a = 0.9$). A numerical solution to the Grad-Shafranov equation is obtained via the solver architecture in the [CoCoNut](#) code (cf. Adsuara et al., 2016) and as initial data for simulations employing the [Einstein Toolkit](#).

- The numerical techniques solving the Grad-Shafranov equation around spinning Kerr BHs may be used with existing infrastructure of numerical PDE solvers, e.g., the [CoCoNut](#) code. (Cerdá-Durán et al., 2009; Adsuara et al., 2016)
- Spacetime initial data for rapidly spinning BHs (high Blandford/Znajek luminosities expected) is tested on the [Carpet](#) grid of the [Einstein Toolkit](#). (Liu et al., 2009)
- We have adapted the evolution routines available for the ET to account for a FF magnetized plasma around spinning BHs implemented as [punctures](#). Our implementation is inspired by previous work on GRMHD using the ET and GRFFE. (Faber et al., 2007; Mösta et al., 2014; Etienne et al., 2017)

Scaling tests need scalable architectures

Parallel tests of GRiFFiN code - way to go!

Tearing mode scaling tests on static (flat) background. *Einstein Toolkit* shows weak scalability (Löffler et al., 2012) on selected large machines.



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Open forum: Let's discuss

Questions. Answers. Remarks. Discussion.

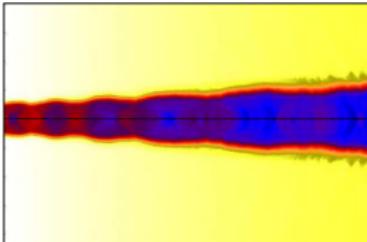
Thank you.



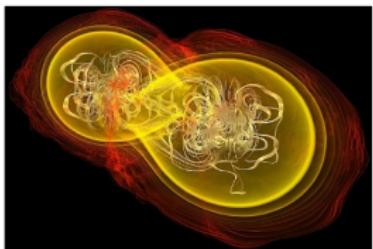
How numerical simulations (may) help us

Examples from CAMAP/Valencia

Neutron star mergers forming a rotating **black hole** with magnetized environment. (Rezzolla et al., 2011; Kiuchi et al., 2014; Ruiz et al., 2016)



Goal: Application of established numerical techniques within other disciplines (e.g., astroeye).

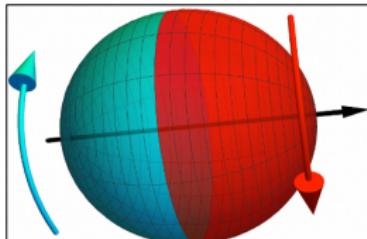


Goal: Probing of specific mechanisms (e.g., *dissipation*, *radiation*) and development of numerical codes/strategies.

Interdisciplinary work, e.g., a joined project of mathematicians, theoretical physicists and optometrists at the *University of Valencia*.

Goal: Access to astrophysical scenarios (e.g., **binary mergers**, gravitational waves, **jets**).

Numerical simulation of **jet** propagation (Mimica et al., 2013).



Relativistic jets are very strong accelerators

Observation of cosmic high energy phenomena

- *Jets* are highly *collimated* outflows from compact objects.
- Very high *Lorentz factors* are possible.
- Both internal and external *shocks* are subject of current studies and simulations.
- *Dissipative* mechanisms occur in interaction with the *interstellar medium*.

Open questions remain mainly in jet launching and collimation.

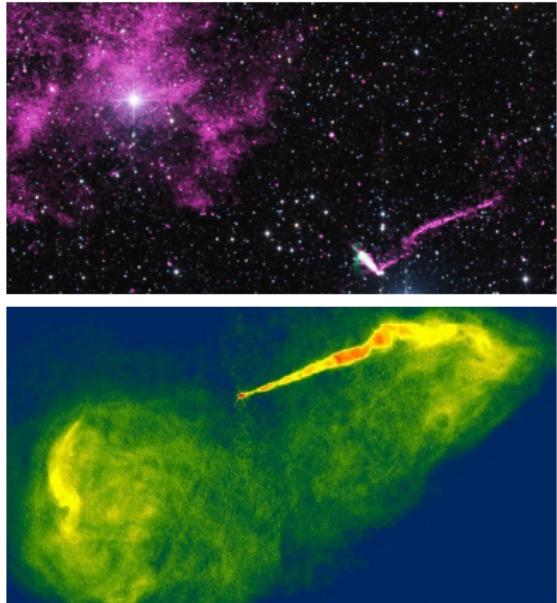
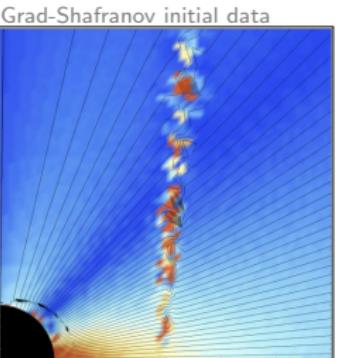


Figure: Top: Pulsar IGR J11014-6103 (Chandra X-ray observatory). Bottom: M87 radio galaxy (National Radio Astronomy Observatory).

Research stages and methods

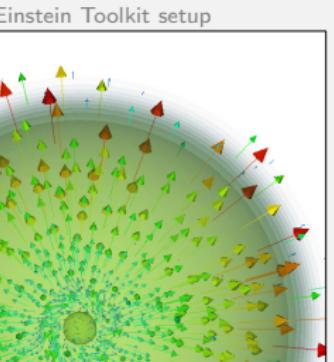
Numerical simulations as astrophysical experiments

Stage I: Theory/Numerics



- Implementation and testing of a numerical procedure for **magnetosphere** data.
- Expand solving schemes towards more **complicated field topologies**.

Stage II: Simulations



- Preparation of initial data for time evolution setups.
- Adaptation of suitable evolution schemes (employing the **Einstein Toolkit**).

Stage III: Evaluation/Feedback

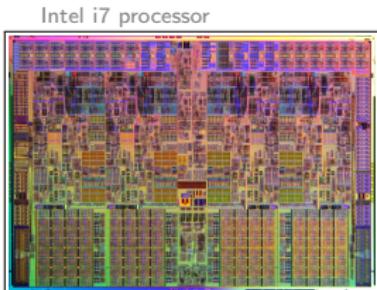
Fortran code segment

```
call Compute_Function_Value(PsiGrid(i,j,k), ITmp)
! Linear operator coefficients >>>>>>>>>
! Linear terms from BZ77 - Simplified in Mathematica
c1(1,1,i,j,k) = (1.0d0*SigmaBL**2.0d0)* &
(2.0d0*bhspinsq*cBL_sq*(bhmass+OmegaTapp*BL_sq)) +&
2.0d0*rBL*((-bhmass)*OmegaTapp**BL_sq)*(2.0d0*bhspinsq*(bhmass+rBL)*OmegaTapp*BL_sq)
c11(1,1,i,j,k) = ((2.0d0*bhmass-rBL)*rBL*bhspinsq*(bhmass**2.0d0*bhmass+rBL+rBL_sq))
c2(1,1,i,j,k) = (32.0d0*(bhspinsq+rBL_sq)*DeltaBL**(-8.0d0*bhspinsq*(3.0d0*bhspinsq**4.0d0*rBL
(5.0d0*bhspinsq**6.0d0*16.0d0**6.0d0*16.0d0
4.0d0*bhspinsq**(-bhspinsq**4.0d0*bhspinsq**4.0d0*theta(j))*bhspinsq**4.0d0*DeltaBL**0
(32.0d0*DeltaBL*SigmaBL**2.0d0)
c22(1,1,i,j,k) = (((2.0d0*bhmass-rBL)*rBL*bhspinsq*(bhspinsq**2.0d0*bhmass+rBL+rBL_sq))
! Non-linear operator coefficients >>>>>>>>
! Non-linear terms from BZ77 - Simplified in Mathematica
if ((i.lt.1).or.(i.gt.m).or.(j.lt.1).or.(j.gt.n)
Source(i,j,k) = Source(i,j,k)
else
Source(i,j,k) = (-i*Bhmass_sq*(-2.0d0*bhspinsq*bhmass_sq*OmegaTapp*DeltaBL**(-1.0d0*rBL**2.0d0))
Source(i,j,k) = Source(i,j,k)
endif
```

- Classify initial data in terms of **stability** and the observation of **jet launching**.
- Understand the role of force-free evolution vs. ideal MHD implementations.

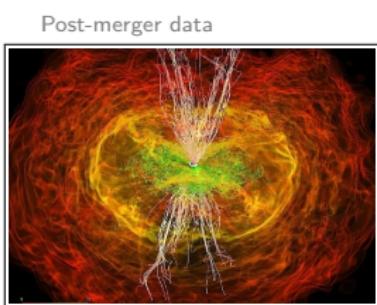
How to contribute to the community?

Key drivers: Complexity and cost reduction



Despite *parallelized codes*, large computational power is required for simulation projects and everyday work:

- Rezzolla et al. (2011) project required ~ 200.000 *core hours*.
- CAMAP used ~ 250.000 core hours in wide-area computer centers the last 4 month (more than 5 million the last 5 years).



Relativistic jets are a post-merger phenomenon, hence, the (violent) merger simulation is not required **if** physical initial data could be used instead:

- Reduction of numerical complexity of the employed code.
- Faster access to relevant simulation episodes (in order to do theoretical astrophysics).

Time evolution of FF electrodynamics I

Comparison of evolution schemes

[1] Full Maxwell's equations evolution

(Komissarov, 2002, 2004, 2007; Paschalidis and Shapiro, 2013)

$$\nabla_\nu F^{\mu\nu} = J^\mu \quad \nabla_\nu {}^*F^{\mu\nu} = 0$$

[2] Energy flow evolution

(McKinney, 2006; Paschalidis and Shapiro, 2013; Etienne et al., 2017)

$$\nabla_\mu T^\mu_\nu = 0 \quad \nabla_\nu {}^*F^{\mu\nu} = 0$$

Augmented system

(Dedner et al., 2002; Palenzuela et al., 2009; Mignone and Tzeferacos, 2010)

$$\begin{aligned} \nabla_\nu \left({}^*F^{\mu\nu} + \left(c_h^2 \gamma^{\mu\nu} - n^\mu n^\nu \right) \psi \right) &= -\kappa_\psi k^\mu \psi \\ \nabla_\nu (F^{\mu\nu} + g^{\mu\nu} \phi) &= J^\mu - \kappa_\phi k^\mu \phi \end{aligned}$$

Augmented system

(Dedner et al., 2002; Palenzuela et al., 2009; Mignone and Tzeferacos, 2010)

$$\nabla_\nu \left({}^*F^{\mu\nu} + \left(c_h^2 \gamma^{\mu\nu} - n^\mu n^\nu \right) \psi \right) = -\kappa_\psi k^\mu \psi$$

The $\text{div}\mathbf{B} = 0$ and $\text{div}\mathbf{D} = \rho$ constraints are ensured by a mixed *hyperbolic/parabolic* correction with the additional scalar potentials ψ and ϕ . In its analogy to the *telegraph equation*, the factor c_h is the finite propagation speed of divergence errors, the constants κ_ψ and κ_ϕ are their damping rate. The above equations are formulated in a *conserved flux formulation*:

$$\partial_t \mathcal{C} + \partial_j \mathcal{F}^j = \mathcal{S}_n + \mathcal{S}_s$$

Time evolution of FF electrodynamics II

Conserved flux formulation (dynamic spacetimes)

[1] Full Maxwell's equations evolution

- Requires **(force-free currents)** (cf. Komissarov, 2011)
- Fluxes derived from **conserved** quantities

$$\mathcal{C} \equiv \gamma \begin{pmatrix} \frac{\psi}{\alpha} \\ \frac{\phi}{\alpha} \\ B^i + \frac{\psi}{\alpha} \beta^i \\ D^i - \frac{\phi}{\alpha} \beta^i \end{pmatrix} \quad \mathcal{F}^j \equiv \gamma \begin{pmatrix} B^j - \frac{\psi}{\alpha} \beta^i \\ - (D^j + \frac{\phi}{\alpha} \beta^i) \\ e^{ijk} E_k + \alpha \left(\textcolor{red}{c_h}^2 \gamma^{ij} - n^i n^j \right) \psi \\ - (e^{ijk} H_k + \alpha g^{ij} \phi) \end{pmatrix}$$

$$\mathcal{S}_n \equiv \begin{pmatrix} -\gamma \alpha \psi \Gamma_{\alpha\beta}^t \left(\textcolor{red}{c_h}^2 \gamma^{\alpha\beta} - n^\alpha n^\beta \right) \\ -\gamma \alpha \phi \Gamma_{\alpha\beta}^t g^{\alpha\beta} - \gamma \rho \\ -\psi \left[\alpha \gamma \Gamma_{\alpha\beta}^i \left(\textcolor{red}{c_h}^2 \gamma^{\alpha\beta} - n^\alpha n^\beta \right) \right] \\ -\gamma \alpha \phi \Gamma_{\alpha\beta}^i g^{\alpha\beta} - \gamma \textcolor{teal}{J}^i \end{pmatrix} \quad \mathcal{S}_s \equiv \begin{pmatrix} -\alpha \gamma \kappa_\psi \psi \\ -\alpha \gamma \kappa_\phi \phi \\ 0 \\ 0 \end{pmatrix}$$

[2] Energy flow evolution

- \mathbf{D} is reconstructed ($\mathbf{D} \cdot \mathbf{B} = 0$)
- Fluxes derived from **primitive** quantities

$$\mathcal{C} \equiv \gamma \begin{pmatrix} \frac{\psi}{\alpha} \\ B^i + \frac{\psi}{\alpha} \beta^i \\ \alpha T^t_i \end{pmatrix} \quad \mathcal{F}^j \equiv \gamma \begin{pmatrix} B^j - \frac{\psi}{\alpha} \beta^i \\ e^{ijk} E_k + \alpha \left(\textcolor{red}{c_h}^2 \gamma^{ij} - n^i n^j \right) \psi \\ \alpha T^i_j \end{pmatrix}$$

$$\mathcal{S}_n \equiv \begin{pmatrix} -\gamma \alpha \psi \Gamma_{\alpha\beta}^t \left(\textcolor{red}{c_h}^2 \gamma^{\alpha\beta} - n^\alpha n^\beta \right) \\ -\psi \left[\alpha \gamma \Gamma_{\alpha\beta}^i \left(\textcolor{red}{c_h}^2 \gamma^{\alpha\beta} - n^\alpha n^\beta \right) \right] \\ \frac{1}{2} \alpha g_{\mu\nu,i} T^{\mu\nu} \end{pmatrix} \quad \mathcal{S}_s \equiv \begin{pmatrix} -\alpha \gamma \kappa_\psi \psi \\ 0 \\ 0 \end{pmatrix}$$

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