Spacetime Discretization Methods for Numerical Relativity

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Current approach to Numerical Relativity

- Accurate modelling of highly dynamical, strong field astrophysical phenomena, such as binary black hole mergers require detailed, large scale numerical simulations. Required, for example, in searching for gravitational waves with LIGO.

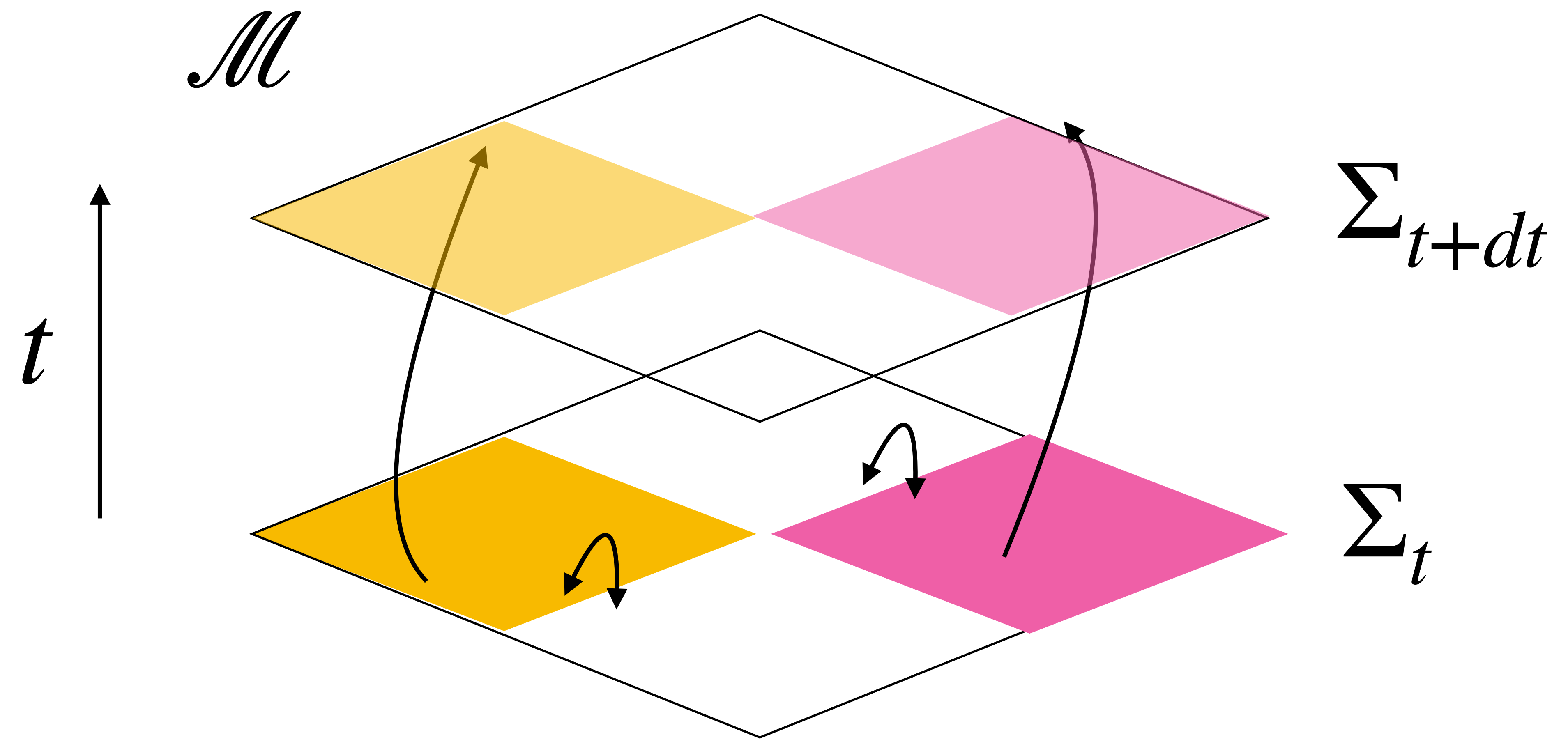
- Successful simulation of a binary black hole merger is quite recent[1]. These simulations are computationally expensive; in many cases, prohibits exploration of the full parameter space and even new physics.

- Most NR codes employ a 3+1 split to decompose Einstein's field equations of general relativity into a Cauchy initial value problem. They employ a finite-difference or spectral discretization in each spatial slice and evolve in time.

- **Parallelism is obtained by dividing the spatial domain into patches** that can be computed in parallel at each time step.

- Patches have time-like boundaries; characteristics cross in both directions. **Requires inter-patch communication at every time step.**

- **Communication is expensive**; in fact, probably the slowest operation in modern HPC systems.



Patches communicate boundary data on each time step.

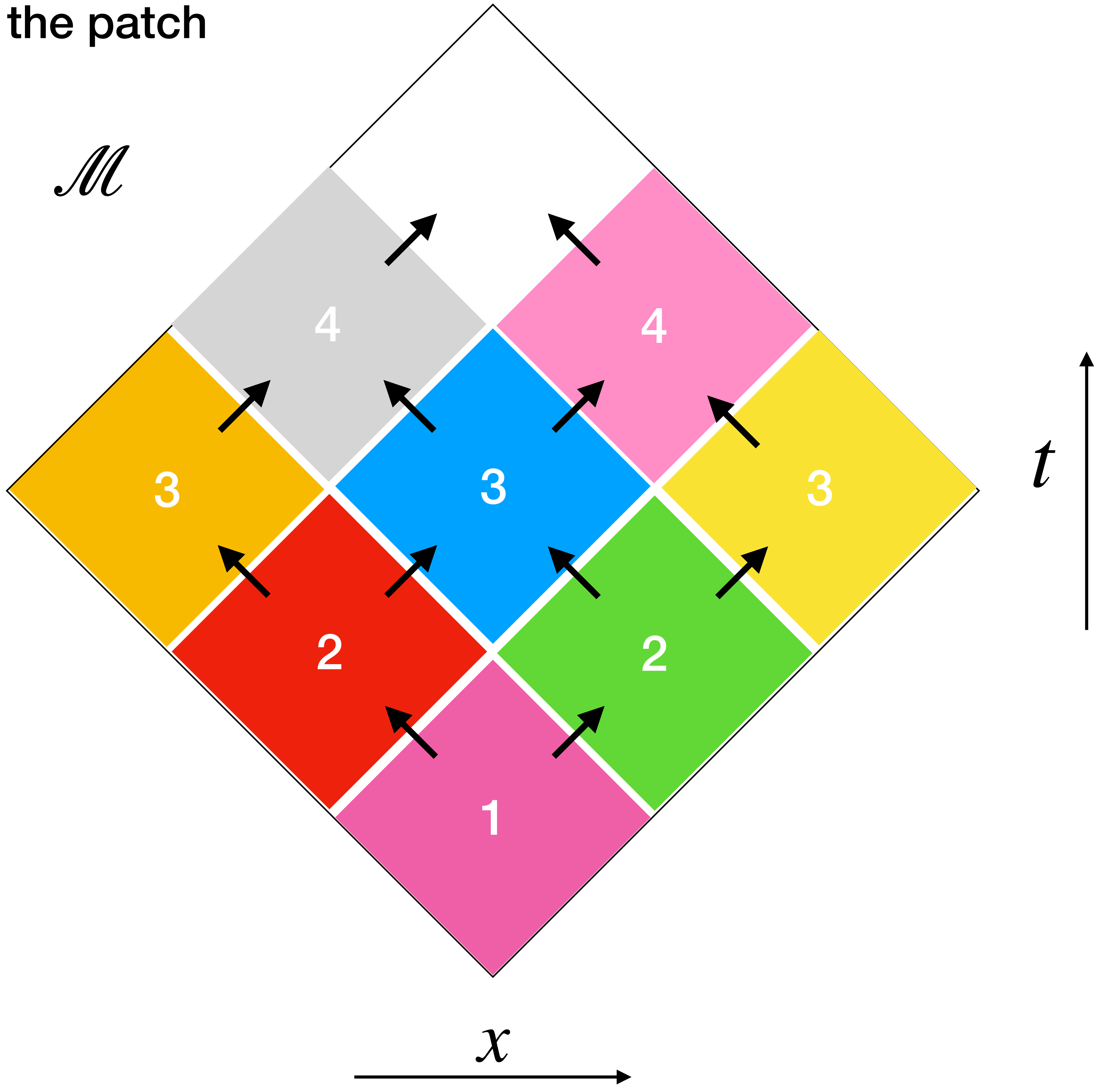
[1]Evolution of Binary Black Hole Spacetimes, Pretorius, Phys. Rev. Lett. **95**, 121101, 2005

Our approach — Spacetime Elements

- **Use a spacetime patch.** Instead of discretizing in space and time, chunk up the manifold in spacetime patches.

- **Removes the need for communication at every time step, if one chooses the patch boundaries wisely; i.e. null or space-like.**

- Spacetime patches only require an initial condition at the incoming patch boundaries, and the results of the computation, i.e. the outgoing patch boundaries, serve as initial conditions for the future patches.



To the right, we show a spacetime discretization of a 1+1 spacetime manifold. The patches are numbered according to causality. Patches with the same number can be solved in parallel.

- **Use Discontinuous Galerkin Finite elements representation.** Provides a **tunable balance** between achieving high **work-efficiency** and increased **parallelism** on current HPC systems.

- A simple back-of-the-envelope calculation shows one could reduce the number of communication steps by a factor of ~40 (probably too optimistic).

Ongoing Research

- Developing computational infrastructure to solve a toy model: the scalar wave equation on curved backgrounds using spacetime discretization methods.

- To get the equations of motion, start with the action, discretize, and then extremize.

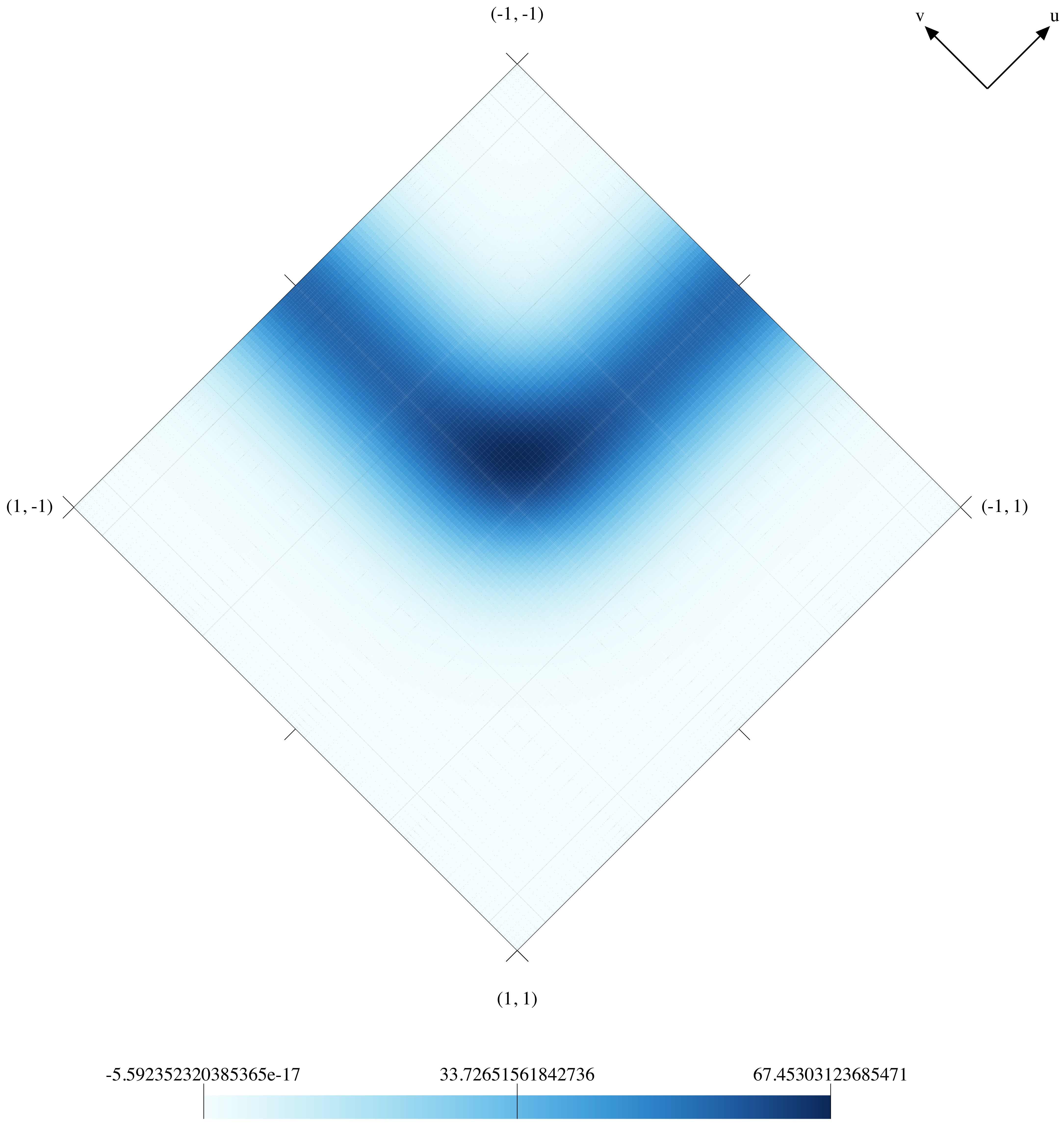
$$S = \int \eta^{\alpha\beta} \nabla_\alpha \phi \, \nabla_\beta \phi \, d^2x$$

$$S^N = \sum_{p,q,k,l}^{N^2} w_p w_q \, \eta^{\alpha\beta}_{pq} \, [D_\alpha]^p_l \, \phi^l \, [D_\beta]^p_m \, \phi^m \longrightarrow S^N = \phi^{\mathrm{T}} \mathbf{L} \, \phi$$

- Using **Futures** in julia

  to asynchronously compute multiple spacetime patches concurrently. Abstracts away the complexity behind explicit message passing between patches.

  Define patch^{n+1} = Future of (patch$^{n}{}_{L}$ + patch$^{n}{}_{R}$)



A spacetime perturbation propagating in 1+1 Minkowski spacetime. Using 16 patches in total, with 20 points along each direction in each patch.

Future Extensions

- Extend the infrastructure to be able to handle higher dimensional problems, concretely 3+1 for classical gravity.

- Use an unstructured mesh — spacetime elements will now be higher order simplexes. Would allow for more flexibility to adapt the grid to the problem. Useful, for example, to study the structure of apparent horizons near merger.

- For unstructured meshes, given a tessellation on an initial hyper surface, the **mesh front can be advanced in time using the tent-pitching algorithm** proposed by Üngör et. al. 2000[2].

- Unstructured grid usually considered to be less efficient due to the overhead of managing mesh connectivity. However, use of higher order polynomials in each element or clubbing multiple elements together can amortize this cost.

- Work with conformally compactified domains. Removes the need for an artificial outer-boundary, and provides a clean way to extract radiation at null infinity.

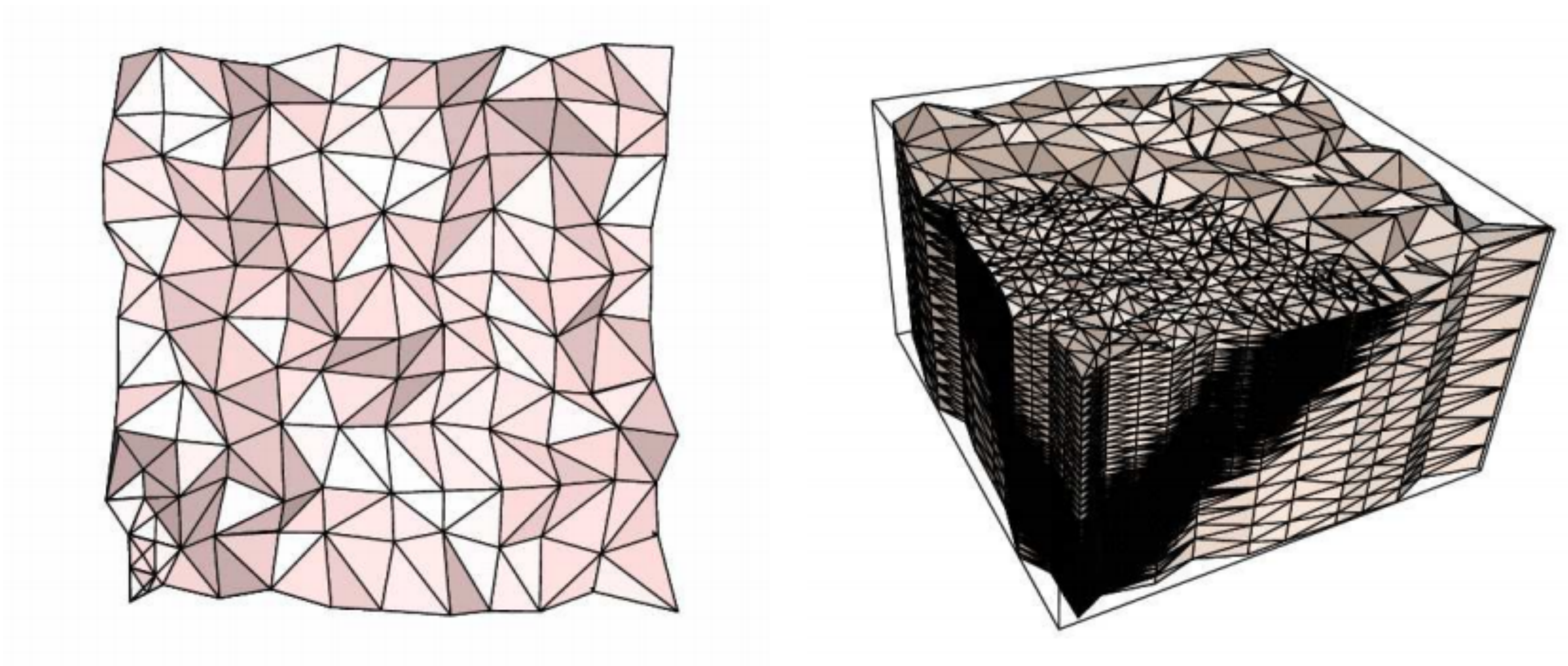


Figure 1.6 from *Shripad Thite, Spacetime meshing for Discontinuous Galerkin methods.* Given a 2D tessellation, the algorithm constructs a tetrahedral spacetime mesh.

[2]Alper Üngör and Alla Sheffer. Tent-pitcher: A meshing algorithm for space-time Discontinuous Galerkin methods