Parallel multigrid for 3D Poisson equation on cylindrical coordinate

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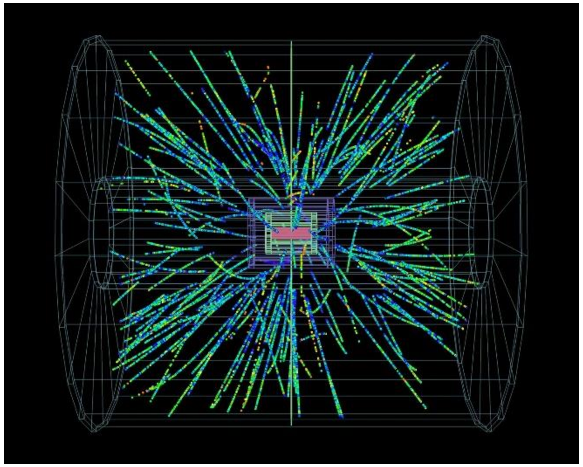
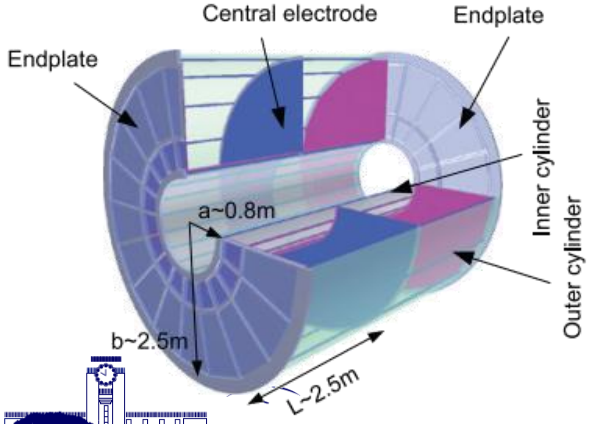
Background



Source: http://public.web.cern.ch/public/features/ALICE-proton-lead.jpg



- Poisson equation often used to model a wide range of phenomena in computational physics
- Regardless the methods, large scale simulation requires a large computational power, thus demand for fast solver
- The observed phenomena used in this research related with high energy particle experiment
- When there is a collision that produces charged particles across the detector, gas ions distort the electron trajectories that will shift the value of place and time detected in the endplate
- To correct the distortion, the most time consuming step is the calculation of poisson equation

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Model

- Equation

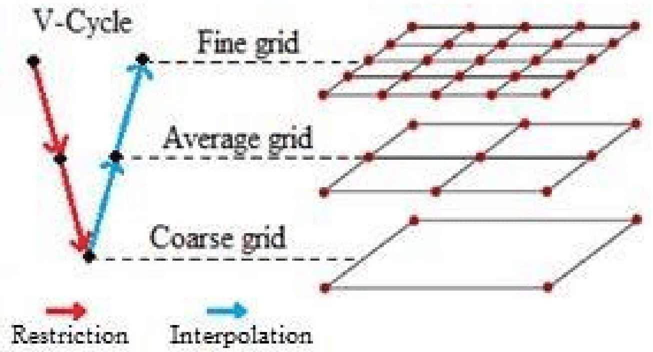
$$\frac{\partial^2 U}{\partial r^2} + \frac{1}{r}\frac{\partial U}{\partial r} + \frac{1}{r^2}\frac{\partial^2 U}{\partial \theta^2} + \frac{\partial^2 U}{\partial z^2} = f(r,\theta,z)$$

- Discretization
  - The 2nd order Finite Difference Method
  - 3D stencil notation
  - $f = \frac{1}{h_r^2}\left[\begin{array}{ccc|ccc|ccc} 0 & 0 & 0 & 0 & \alpha & 0 & 0 & 0 & 0 \\ 0 & \rho & 0 & 1-\omega & -2(1+\alpha+\rho) & 1+\omega & 0 & \rho & 0 \\ 0 & 0 & 0 & 0 & \alpha & 0 & 0 & 0 & 0 \end{array}\right] u$
  - Where:
  - $\rho = \frac{h_r^2}{h_z^2}, \alpha_i = \frac{h_r^2}{r_i^2 h_\theta^2}, \omega_i = \frac{h_r}{2r_i}$

Multigrid

Two grid method:

1. Pre-smoothing at fine grid (h), compute u^h as solution of $A^h u^h = f^h$
2. Compute residu: $r^h = f^h - Au^h$
3. Restrict the residu to coarse grid (2h): $r^{2h} = \mathcal{R}_h^{2h} r^h$
4. Solve residual equation at coarse grid for the error: $A^{2h} e^{2h} = r^{2h}$
5. Prolongate/interpolate error from coarse to fine grid: $e^h = \mathcal{I}_{2h}^h e^{2h}$
6. Compute the next approximation by: $u^h = u^h + e^h$
7. Post=smoothing at fine grid: $A^h u^h = f^h$



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Parallelization Plan

- Smoother
  - Block Red Black Gauss Seidel (Kawai 2012, 2015)
    - Entire grid divided into multiple blocks
    - Color ordering applied to each block
    - The Gauss Seidel applied to the block in parallel
  - Iterate GS in each block more than once to improve convergence
  - Increase vectorization by applied loop splitting method