Development of a scalable spectral element solver for nonlinear water wave simulations

Nickolas Foskolos - DTU - Denmark

Problem background

Laplace problem

Free surface BC

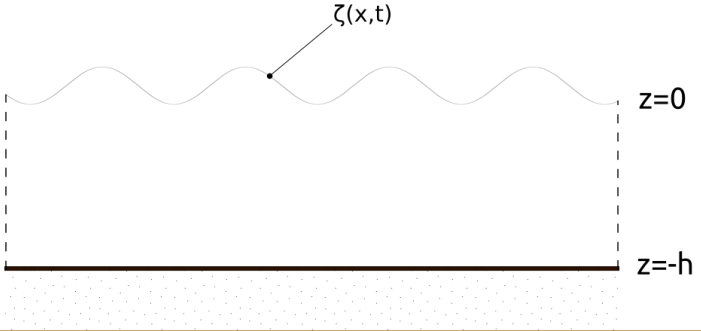
$$\phi = \tilde{\phi}, \quad z = \zeta(x,t)$$

Domain Laplace equation

$$\nabla^2\phi + \vartheta_{zz}\phi = 0, \quad -h \leq z < \zeta(x,t)$$

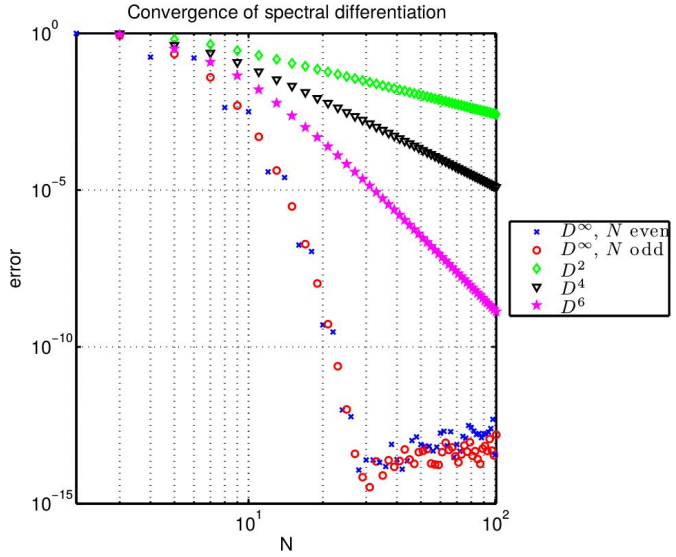
Bottom BC

$$\vartheta_z\phi + \nabla h \cdot \nabla\phi, \quad z = -h$$



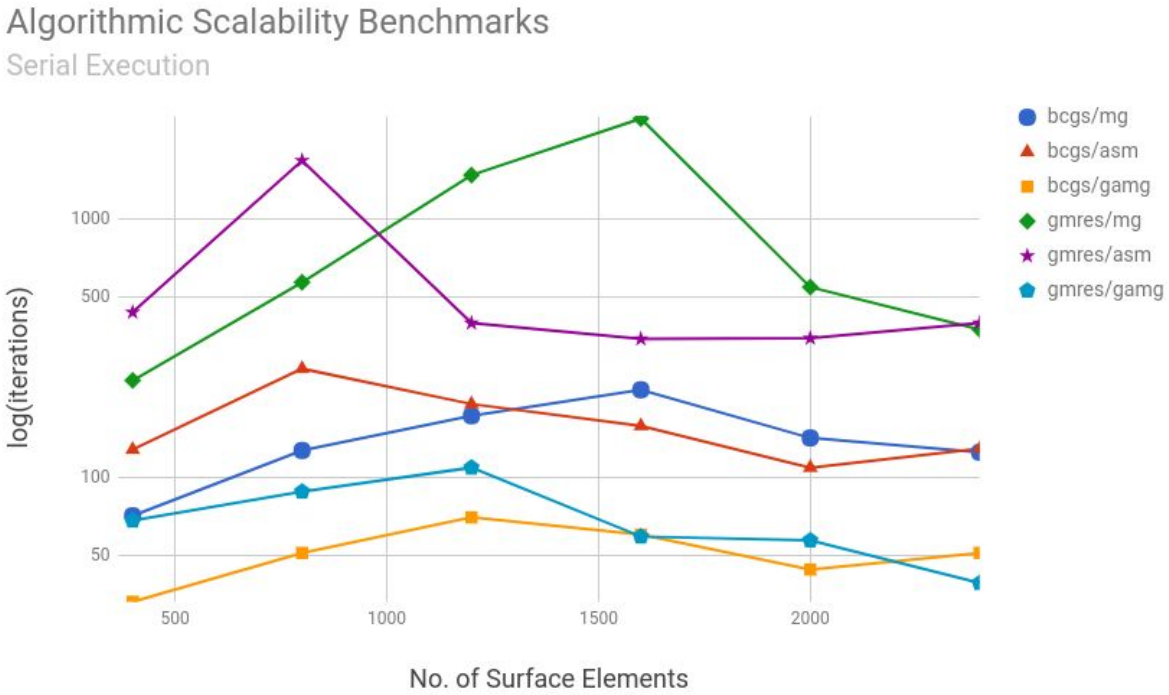
Spectral element discretization

$$\frac{du}{dx} = Du, \quad D_{ij} = \begin{cases} -\frac{1}{2}(-1)^{i-j}\left[\sin\left(\frac{\pi(i-j)}{N+1}\right)\right]^{-1} & , i \neq j \\ 0 & , i = j \end{cases}$$



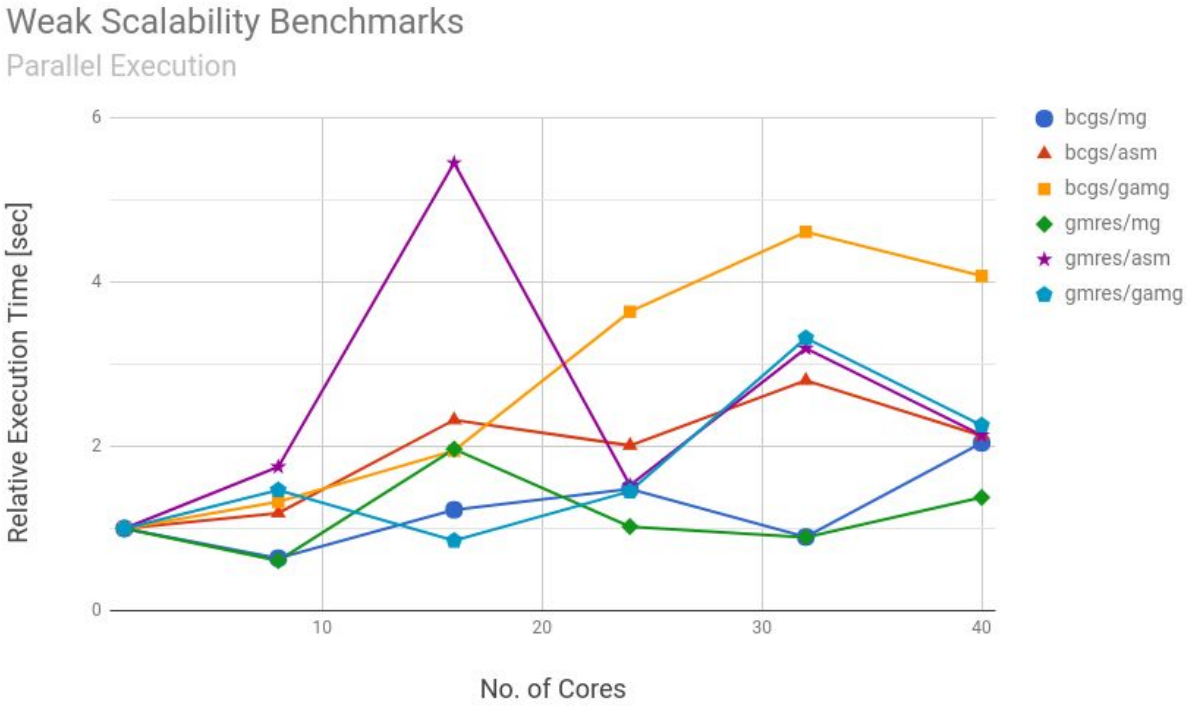
Serial scalability tests

- First step to ensure algorithmic scalability
- Using PETSc for parallel implementation
- Testing different solver/pc combinations



Parallel scalability tests

- Testing up to 40 cores
- Problem is bound by memory limitations
- Best performance obtained for a combination of GMRES with Multigrid preconditioning



Conclusions and future work

- Validation against the exact solution
- Good algorithmic and weak scalability are achieved through the GMRES/MG solver

In the future:

- Handle the matrix assembly in parallel to surpass memory limitations
- Integrate the 2D solver in our spectral element framework
- Perform tests on more difficult geometries, including solid objects in the domain