

PyGBe - LSPR

**Python and GPU Boundary-integral
solver for electrostatics.**

Natalia C. Clementi

The George Washington University



<https://github.com/barbagroup/pygbe>



nclementi@gwu.edu

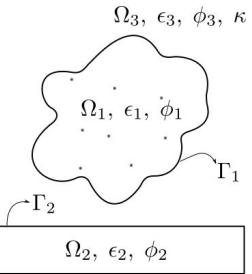
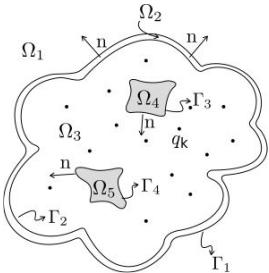


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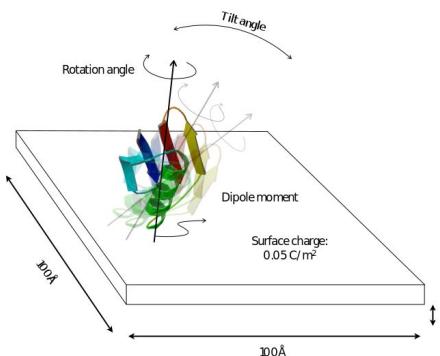
We use PyGBe for...

Poisson-Boltzmann equation of the [implicit-solvent](#) model in integral form.

Solvation energies for proteins modeled with any number of dielectric regions

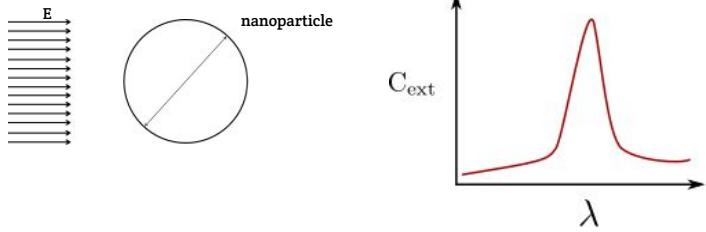


Protein surface interaction

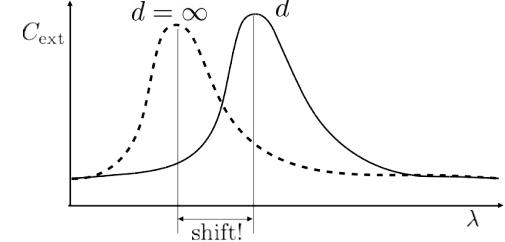
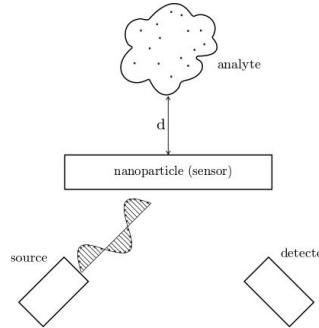


Probing protein orientation near charged nanosurfaces.

Localized surface plasmon resonance of nanoparticles.



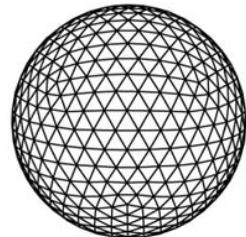
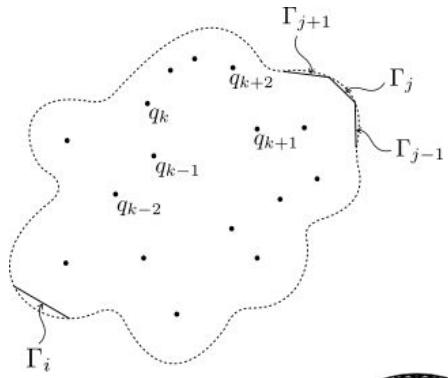
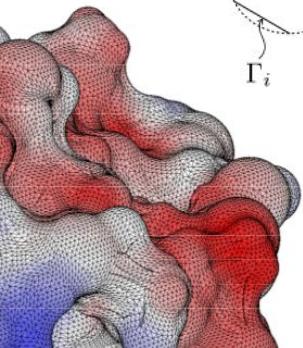
Compute the extinction cross section of scatterers that are much smaller than the incident wavelength



How do we do it ...

Boundary Element Method (BEM)

$$\begin{array}{ccc} \text{PDE} & \xrightarrow{\hspace{1cm}} & \text{BIE} \\ \nabla^2 \phi = 0 & & \phi = \oint_{\Gamma} G \frac{\partial \phi}{\partial \mathbf{n}} dS - \oint_{\Gamma} \phi \frac{\partial G}{\partial \mathbf{n}} dS \end{array}$$



Biomolecular Electrostatics

$$\begin{bmatrix} \frac{1}{2} + K_L^\Gamma & -V_L^\Gamma \\ \frac{1}{2} - K_L^\Gamma & \frac{\epsilon_1}{\epsilon_2} V_L^\Gamma \end{bmatrix} \begin{bmatrix} \phi_{1,\Gamma} \\ \frac{\partial}{\partial \mathbf{n}} \phi_{1,\Gamma} \end{bmatrix} = \begin{bmatrix} \sum_{k=0}^{N_q} \frac{q_k}{4\pi |\mathbf{r}_\Gamma - \mathbf{r}_k|} \\ 0 \end{bmatrix}$$

Localized Surface Plasmon Resonance $\epsilon = \epsilon(\omega)$

$$\begin{bmatrix} \frac{1}{2} + K_L^\Gamma & -V_L^\Gamma \\ \frac{1}{2} - K_L^\Gamma & \frac{\epsilon_1}{\epsilon_2} V_L^\Gamma \end{bmatrix} \begin{bmatrix} \phi_{1s,\Gamma} \\ \frac{\partial}{\partial \mathbf{n}} \phi_{1s,\Gamma} \end{bmatrix} = \begin{bmatrix} Q \\ \frac{\epsilon_2 - \epsilon_1}{\epsilon_2} \frac{\partial \phi_i}{\partial \mathbf{n}} \end{bmatrix}$$

$$V_{L,ij}^\Gamma = \int_{\Gamma_j} G_L(\mathbf{r}_{\Gamma_i}, \mathbf{r}_{\Gamma_j}) d\Gamma_j$$

$$K_{L,ij}^\Gamma = \int_{\Gamma_j} \frac{\partial}{\partial \mathbf{n}} [G_L(\mathbf{r}_{\Gamma_i}, \mathbf{r}_{\Gamma_j})] d\Gamma_j$$

How do we do accelerate it ...

BEM

Problem: Dense matrix, Gaussian elimination, $O(N^3)$ computations.

Integrals are approximated with gauss quadrature rules converting the mat-vec in an N body problem

GMRES

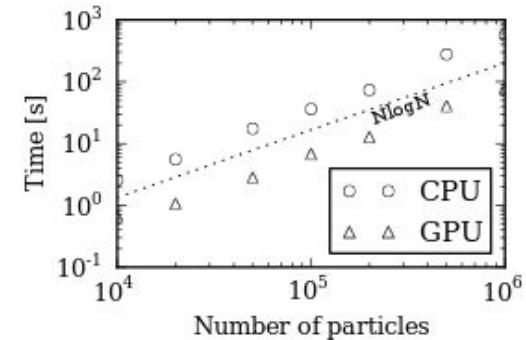
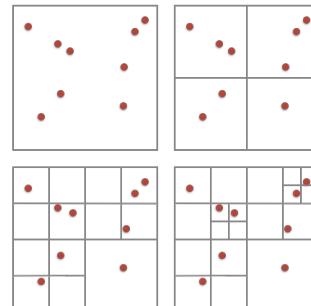
Krylov subspace iterative solver. It reduces computations to $O(N^2)$ per iteration.

Barnes and Hut Treecode

Approximate far-away source-target interactions with a multipole expansion using Taylor expansions.

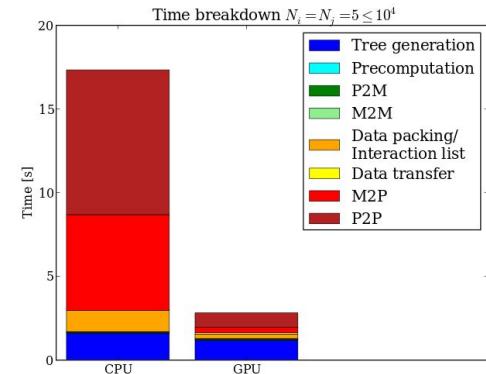
Potential due to point-sources (gauss points) of mass is calculated on target (collocation points) locations.

Reduces computations from $O(N^2)$ to $O(N \log N)$.

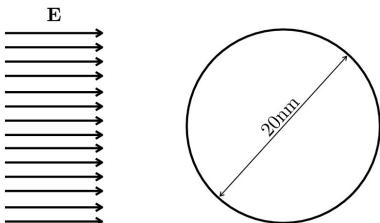


GPU

Consuming parts of the treecode M2P and the P2P interactions.



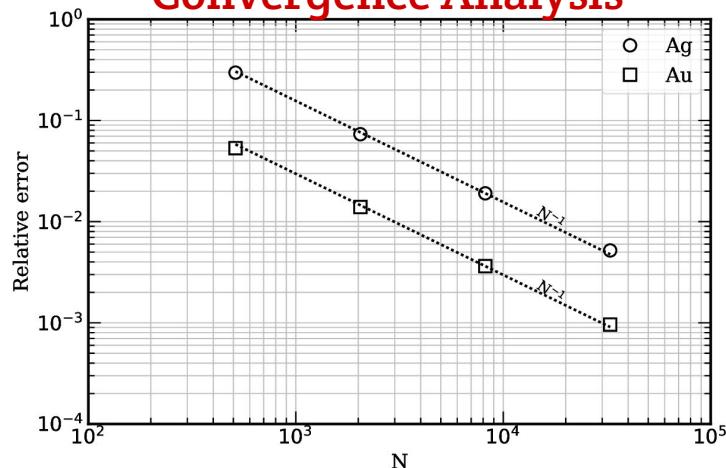
LSPR single NP verification



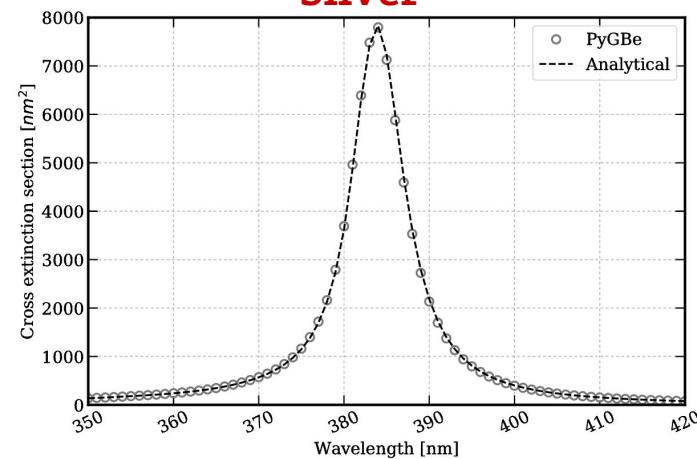
GPU - NVIDIA K40 (time per run)

N	% err_Ag	time_Ag [s]	% err_Au	time_Au [s]
512	29.73	2.35	5.328	2.10
2048	7.32	8.02	1.396	7.19
8192	1.91	62.83	0.363	53.33
32768	0.52	350.63	0.096	298.13

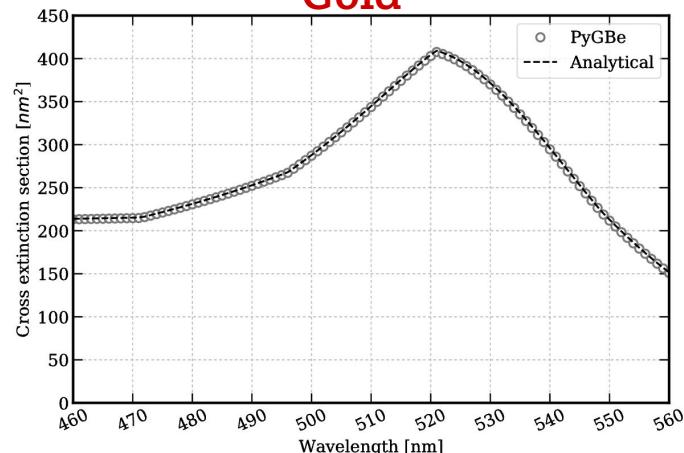
Convergence Analysis



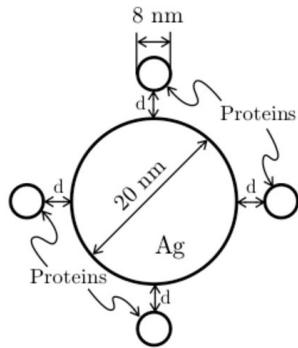
Silver



Gold



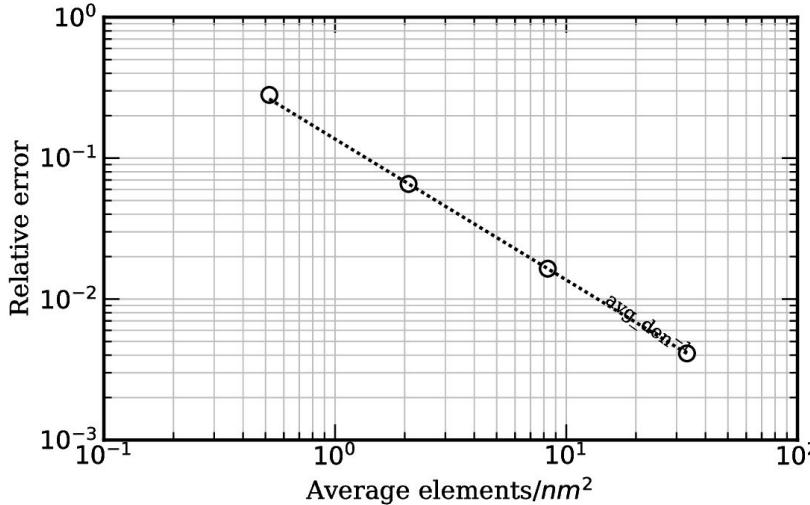
LSPR sensor response POC



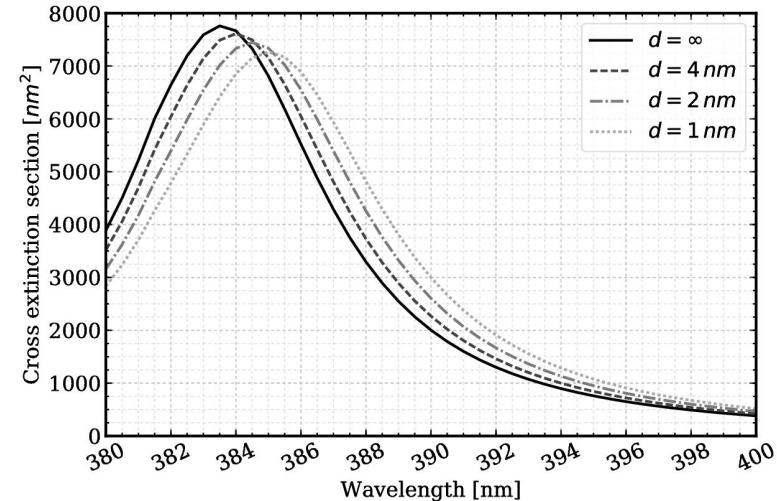
GPU - NVIDIA K40 (time per run)

avg_dens	% error	time [s]
0.52	28.09	24.72
2.08	6.56.	137.70
8.32	1.64	649.50
33.26	0.41	3695.76

Convergence Analysis



Silver - BSA



References:

- Cooper, C.D and Barba, L.A. (2016), "Poisson–Boltzmann model for protein–surface electrostatic interactions and grid-convergence study using the PyGBe code," *Computer Physics Communications*, **202**: 23–32, [doi: 10.1016/j.cpc.2015.12.019](https://doi.org/10.1016/j.cpc.2015.12.019), [arXiv:1506.03745](https://arxiv.org/abs/1506.03745)
- Cooper, C.D, Bardhan, J.P. and Barba, L.A. (2014), "A biomolecular electrostatics solver using Python, GPUs and boundary elements that can handle solvent-filled cavities and Stern layers," *Computer Physics Communications*, **185**(3): 720–729, [doi: 10.1016/j.cpc.2013.10.028](https://doi.org/10.1016/j.cpc.2013.10.028), [arxiv:1309.4018](https://arxiv.org/abs/1309.4018)
- Cooper, C.D and Barba, L.A. (2016), "Poisson–Boltzmann model for protein–surface electrostatic interactions and grid-convergence study using the PyGBe code," *Computer Physics Communications*, **202**: 23–32, [doi: 10.1016/j.cpc.2015.12.019](https://doi.org/10.1016/j.cpc.2015.12.019), [arXiv:1506.03745](https://arxiv.org/abs/1506.03745)
- Cooper, C.D, Clementi, N.C. and Barba, L.A. (2015), "Probing protein orientation near charged nanosurfaces for simulation-assisted biosensor design," *Journal of Chemical Physics*, **143**: 124709 [doi: 10.1063/1.4931113](https://doi.org/10.1063/1.4931113), [arXiv:1503.08150v4](https://arxiv.org/abs/1503.08150v4).
- Cooper Villagran, Christopher D. (2015), Biomolecular electrostatics with continuum models: a boundary integral implementation and applications to biosensors, Boston University PhD Thesis. <https://open.bu.edu/handle/2144/15650>
- Natalia C. Clementi, Christopher D. Cooper, Gilbert Forsyth and Lorena A. Barba (2017), PyGBe on Localized Surface Plasmon Resonance (LSPR), [10.6084/m9.figshare.4983311.v3](https://doi.org/10.6084/m9.figshare.4983311.v3)