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# DNS of a supersonic spatially-developing turbulent boundary layer

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## Abstract

Spatially-developing turbulent boundary layers (SDTBL) are very challenging to numerically model by virtue of appropriate and realistic turbulent inflow information needed. In fact, the problem becomes harder if the idea is to account for compressibility effects, i.e. high Mach-number flows. Direct Numerical Simulation (DNS) is a promising numerical tool that resolve all scales according to the analyzed problem. In this study, DNS of turbulent spatially-developing boundary layers in the supersonic regime is performed using the standard Smagorinsky model over an isothermal plate at a Mach number of 2.5. The code used (PHASTA) is based on a Finite-Element Approach and has been parallelized on MPI C++/Fortran90-95.

## Introduction

Though most flows encountered in nature and in aerospace applications are turbulent or partially so, turbulence remains one of the most elusive subjects in aeronautics. There is no general turbulence theory or model. With the addition of compressibility the turbulence problem becomes even more complex [1-5]. With the rapid increase of computing power in recent decades, especially the fast development of highperformance computing (HPC) platforms, modern computational fluid dynamics (CFD) technique is now playing a much more important role in aerodynamic researches. In particular, Direct Numerical Simulation (DNS) and Large-Eddy Simulation (LES) become effective tools for turbulence mechanism researches [6-13] . One of the difficulties that arises when comparing experimental results with numerical simulations is that the large Reynolds numbers achieved in the experiments are not reproducible in the simulations because of the prohibitive computational cost that would be required to resolve all the scales present in the flow.

In this study, Direct Numerical Simulation (DNS) of turbulent spatially-developing boundary layers in the supersonic regime is performed over an isothermal plate at a Mach number of 2.5 with the purpose of assessing flow compressibility and the Smagorinsky model performance on low/high order flow statistics and on the dynamics of coherent structures of Zero Pressure Gradient (ZPG) flows.

# Methodology

The details of the current simulation are provided in this section. One difficulty in performing compressible turbulent boundary layer simulations is that the streamwise direction is inhomogeneous. Furthermore it necessitates use of a long entrance length for the flow to adjust from artificial inflow conditions.

Capturing the physics of turbulent spatially developing boundary layers by using DNS is not a trivial task, due to the following reasons: i) high mesh resolution required in order to resolve the smallest turbulence scales (Kolmogorov scales), ii) the computational box must be large enough to appropriately resolve the influence of the turbulent "superstructures" located in the outer region of the boundary layer, iii) realistic time-dependent inflow turbulent conditions (instantaneous velocity, temperature and pressure) must be prescribed. Figure 1 shows the schematic of the computational domain in order to simulate spatially-developing boundary layers in the incompressible, supersonic and hypersonic regimen [1].

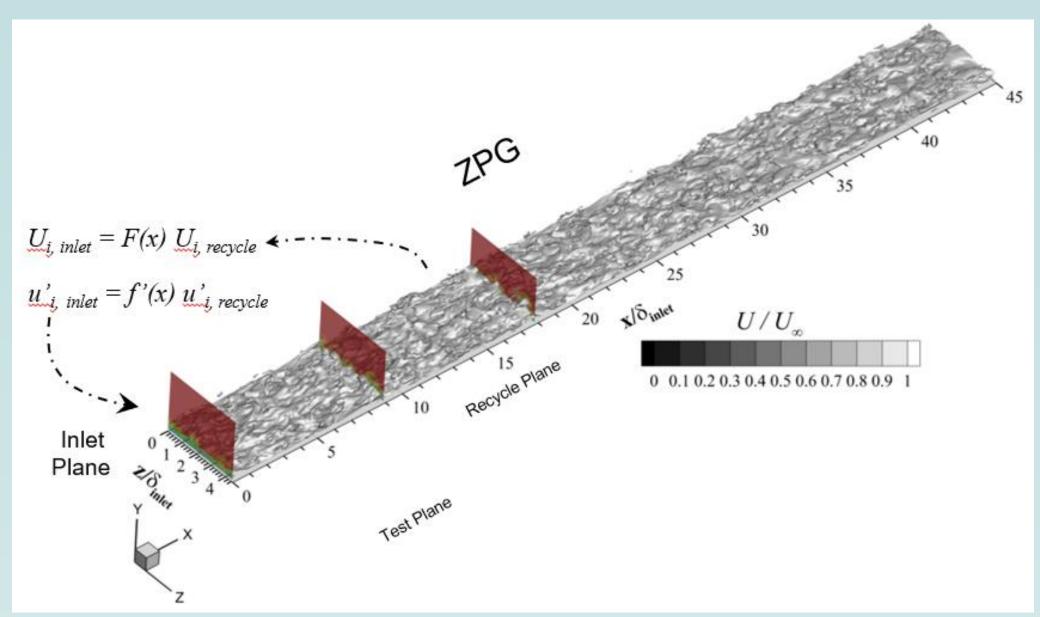


Figure 1: Schematic of the spatially-developing boundary layer with Inlet [1].

Flow Solver: To successfully perform the proposed DNS, a highly accurate, very efficient, and highly scalable CFD solver is required. PHASTA is an open-source, parallel, hierarchic (2nd to 5th order accurate), adaptive, stabilized (nite-element) transient analysis tool for the solution of compressible or incompressible flows.

Boundary Conditions: At the wall, the classical noslip condition is imposed for velocities. Isothermal wall is assumed for the thermal field. For the compressible flow cases, the ratio Tw/Tinf is 2.25, where Tw is the wall temperature and Tinf is the freestream temperature. The lateral boundary conditions are handled via periodicity.

Inflow Generation: In this study, we will make use of the inflow generation method devised by Araya and K. Jansen (2015) [1] which is a modified version of the original rescaling-recycling method. The rescaling method is based on scaling laws of turbulent boundary layers. The inflow is generated by rescaling the flow field at a downstream station and reintroducing it at the upstream inlet. It can be easily implemented to yield a spatial simulation and works very well with little or no transient near the inlet boundary.

## Results and Discussion

#### A. Mean Flow Analysis

The mean streamwise velocity in wall units is shown in figure 2, where the Van Driest transformation (eq. 1) is employed for the supersonic flow data. On the plot we have included the linear sub-layer relation, U+ = y+, the standard log-law, the Spalding velocity profile (eq. 2) and a composite profile that consists of Reichardt's inner layer profile and Finley's wake function (eq. 3) [11]

$$U_c = \int_0^U (\overline{T_w}/\overline{T})^{1/2} dU \tag{1}$$

$$y^{+} = u^{+} + 0.1108\{e^{0.4u^{+}} - 1 - 0.4u^{+} - (0.4u^{+})^{2}/2! - (0.4u^{+})^{3}/3! - (0.4u^{+})^{4}/4!\}$$
 (2)

$$\frac{U_c}{u_\tau} = \frac{1}{\kappa} ln \left( 1 + \kappa \frac{y u_\tau}{\nu_w} \right) + C_1 \left[ 1 - e^{\frac{-y u_\tau}{\eta_1 \nu_w}} - \left( \frac{y u_\tau}{\eta_1 \nu_w} \right) e^{\frac{-y u_\tau b}{\nu_w}} \right] \\
+ \frac{1}{\kappa} \left[ \left( \frac{y}{\delta} \right)^2 - \left( \frac{y}{\delta} \right)^3 + 6\Pi \left( \frac{y}{\delta} \right)^2 - 4\Pi \left( \frac{y}{\delta} \right)^3 \right] \tag{3}$$

It is clearly seen in figure 2 that the mean velocity distribution calculated by DNS agrees with the linear relation in viscous sublayer and the log-law in the logarithmic layer.

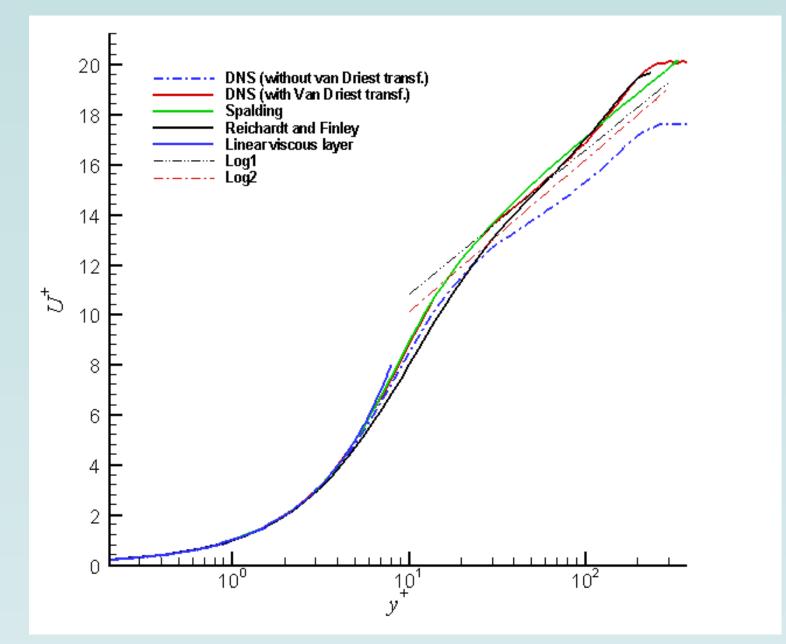


Figure 2: Van Driest transformed velocity in wall units.

Determining the location of the inertial subrange, widely known as the log region, can be done through a log law diagnostic function. The log diagnostic function is implemented as a central difference discretization. The results are plotted in Figure 3.

The log diagnostic function confirms the location of the log region in the range y+=[40; 50] which is consistent with the local minima across the different data and models being studied.

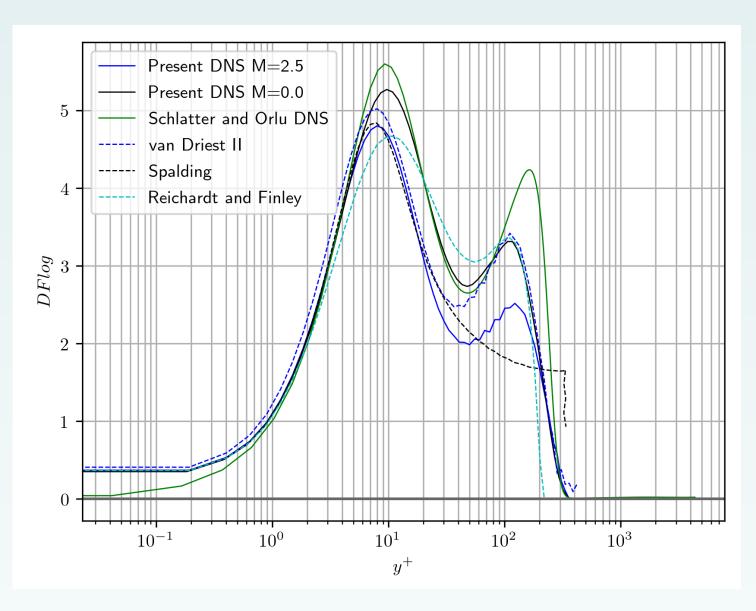


Figure 3: Log law diagnostic function

### B. DNS Validation

Present DNS shows a good agreement with the Walz's equation as well as with experimental data at similar supersonic Mach numbers. The results are plotted in Figure 4.

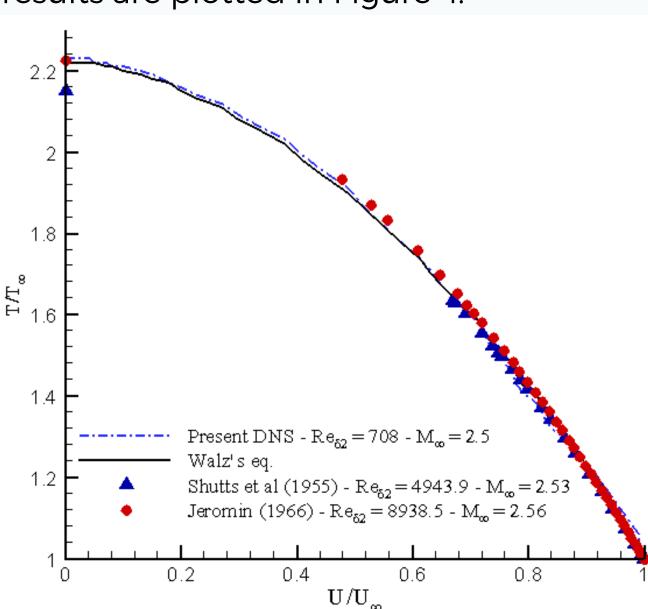


Figure 4: Mean streamwise velocity and temperature.

For the purpose of validation, we compare the simulations with available experimental data [14, 15] and assess the accuracy of the DNS database. Figures 5 plots the mean flow for the DNS and experimental data. There is good agreement among the data, with a small difference near the wall.

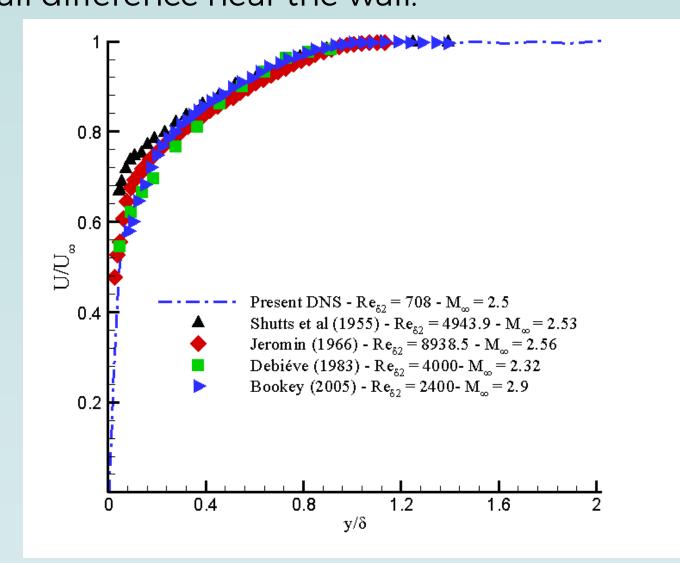


Figure 5: Comparison between DNS and experimental data [14, 15] . Normalized values of mean velocity.

## Conclusion

We perform direct numerical simulations of turbulent boundary layers with freestream Mach number 2.5. The Reynolds number of the simulation was Re = 708. Comparison with available experiments (i.e. those of Debieve and Bookey) suggest that the current simulation provides an accurate description of a compressible turbulent boundary layer.

The DFlog and DFpow functions confirm the location of the log region in the range y+=[40; 50] which is consistent with the local minima across the different data and models being studied.

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